CHOICE BEHAVIOR IN A CUED-RECOGNITION TASK

BY

JAMES T. TOWNSEND

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INTRODUCTION

Several current models of behavior in signal detection and recognition experiments represent the observer's choice on each trial as jointly determined by two distinct hypothetical mechanisms (Atkinson, Carterette, and Kinchla, 1962; Luce, 1963; Swets, Tanner, and Birdsall, 1961). The first mechanism characterizes the effects of psychophysical variables by defining a set of hypothetical sensory states, one of which is activated on each trial. This mechanism is referred to as the sensory process. The second mechanism, the decision process, is affected by learning variables such as the relative frequency of the various signal events on previous trials. The decision process is usually conceived as converting the currently activated sensory state into an overt decision through application of a bias mechanism.

Although the starting point of some theories (and experiments to test them) is the existence of the appropriate sensory and decision processes (e.g., Fox, 1953), others have emphasized the formal similarity between choice behavior in detection tasks and choice behavior in simple prediction experiments (Atkinson, Bower, and Crothers, 1965; Bush, Luce, and Rose, 1964). In fact, models developed in the latter context for simple detection experiments usually reduce to models appropriate for probability learning under certain limiting conditions.

Probabilistic discrimination learning is an extension of probability learning in which every trial is initiated by one of a set of cues, each with a distinct probability distribution over the set of possible outcomes (Popper and Atkinson, 1958; Atkinson, Bogartz, and Turner, 1959).

If, given a particular cue, not all the possible outcome events are equally likely, that cue is said to be correlated with the outcome. Thus, each cue is associated with its own non-contingent reinforcement schedule and a subject can learn to use the cue-outcome correlations to help him make an outcome prediction on each trial.

Just as a simple recognition or detection situation is formally similar to a simple probability learning experiment, so we may develop a psychophysical analogue to the probabilistic discrimination experiment. The resulting paradigm, which we refer to as a cued-detection or cued-recognition task, associates with each of a set of cues a particular probability distribution over the set of signal events. In addition, feedback corresponding to the outcome in the prediction experiments usually concludes each trial by informing the subject which signal event actually occurred on that trial. The results of probabilistic discrimination learning experiments suggest that correlated cues should come to control behavior in a cued-detection or recognition task; i.e., a subject will come to hold several response biases simultaneously, with the effective bias on a given trial being determined by the cue on that trial.

This possibility was recently investigated in the context of an auditory two-interval forced-choice detection task involving three visual cues (Kinchla, Townsend, Yellott, and Atkinson, 1966). The cues were shown to have the predicted effect on the subjects' response behavior and the results generally supported a finite-state detection model suggested by Atkinson and Kinchla (1965).

In this paper, cued-recognition behavior is studied in an experimental setting developed by Estes and Taylor (1964). A model that can be interpreted as a generalization of the Atkinson-Kinchla detection model (Atkinson and Kinchla, 1965; Atkinson, 1963) and as having close ties with the Estes serial-processing model (Estes and Taylor, 1964; Estes and Taylor, 1965) is presented and applied to the experiment. It will be seen that under rather general assumptions about the bias mechanism, the experimental results stringently delimit those cases of the general model that can explain the data.

General Characterization of Experimental Procedure

The present study deals with a visual recognition situation in which one of two types of stimuli is briefly exposed on every trial, and the subject's task is to respond by indicating the stimulus type that occurred. We shall refer to the two types of stimuli as S_1 and S_2 and to their related responses as A_1 and A_2, respectively. A reinforcement event that informs the subject which stimulus type was presented terminates the trial. The information event that denotes an S_1 occurrence will be called an E_1, and the event that denotes an S_2 occurrence will be called E_2. In the experiment reported here, correct information was always given to the subjects.

Each stimulus is in the form of a matrix and consists of a fixed number, $D - 1$, of consonant letters from the English alphabet, plus one of two other symbols (which are not drawn from the English alphabet). Thus, each stimulus display contains D symbols. We shall call the consonants noise symbols and refer to any member of this class of symbols as Z_0. In addition, we shall designate the other symbol present in the display as a signal symbol or simply a signal. It is the signal embedded in the arrangement which specifies the stimulus type. Denoting one of these signals as Z_1 and the other as Z_2, we specify an S_1 by the presence of a Z_1 in the display and an S_2 by the presence of a Z_2 in the display. Thus an S_i may be thought of as a stimulus display consisting of a signal Z_i embedded in an array of Z_0 symbols.

In the experiment to be reported, Z_1 was a circle with a vertical bar, Φ, and Z_2 was a circle with a horizontal bar, Θ.

A feature of the psychophysical task under consideration is that every trial is initiated by one of four cues; in the present case, one of four differently colored lights. Let us denote these cue events C_1, C_2, C_3, and C_4. Each cue has an equal likelihood of occurrence; however, every cue is associated with a different probability distribution for the two types of stimulus presentation, S_1 and S_2. Let the probability of an S_1 presentation, conditional upon the initiation of the trial by cue C_h, be referred to as

$$P(S_1|C_h) = \gamma_h \qquad (h = 1,2,3,4) \ .$$

If $\gamma_1 = \gamma_2 = \gamma_3 = \gamma_4$, the cues are uncorrelated with the signal events; otherwise the schedule is said to be cue dependent or correlated.

In the cued-recognition situation specified above, the following variables can be manipulated: (a) physical parameters of the stimulus displays such as stimulus exposure duration, (b) presentation schedule of S_1 and S_2 for each of the cues, and (c) the outcome structure that specifies the payoffs associated with correct and incorrect responses. The theory that will be developed will describe performance as a function of variables relating to (a) and (b) above, but the experiment reported here involved only manipulation of the presentation schedules associated with the various cue events. The notation developed above may be summarized in the following glossary:

C_h: one of the four cues that initiate each trial $(h = 1,2,3,4)$.

S_i: stimulus type i contains one signal, Z_i and a set of Z_0 symbols of size $D - 1$.

A_j: response by which the subject indicates an occurrence of stimulus S_j $(j = 1,2)$.

E_k: feedback event informing the subject that stimulus S_k occurred $(k = 1,2)$.

Z_0: a noise symbol.

Z_1: signal 1: specifies an S_1 stimulus.

Z_2: signal 2: specifies an S_2 stimulus.

D : the number of symbols in each stimulus display.

γ_h: probability that an S_1 is displayed following initiation of the trial by C_h $(h = 1,2,3,4)$.

The events of a trial occur in the following sequence:

a) Presentation of cue C_h $(h = 1,2,3,4)$.

b) Brief exposure of stimulus S_i $(i = 1,2)$.

c) Subject makes response A_j $(j = 1,2)$.

d) Trial terminates with information event E_i.

In the present experiment the subject was instructed to make a correct response as often as possible, and each trial terminated with an information event which told him whether he was correct or incorrect. There were no monetary payoffs or penalties for correct or incorrect responses.

The major dependent variable is the probability of an A_j response, given that stimulus S_i occurred following C_h. This quantity is denoted as $P(A_j|S_iC_h)$. The subject's performance on a trial initiated by C_h can be described by the stochastic matrix

$$P^{(h)} = \begin{array}{c c} & \begin{array}{cc} A_1 & A_2 \end{array} \\ \begin{array}{c} S_1 \\ S_2 \end{array} & \begin{bmatrix} P(A_1|S_1C_h) & P(A_2|S_1C_h) \\ P(A_1|S_2C_h) & P(A_2|S_2C_h) \end{bmatrix} \end{array} .$$

The reader should note that the probabilities $P(A_1|S_1C_h)$ and $P(A_1|S_2C_h)$ completely specify the matrix $P^{(h)}$, which we shall refer to as the performance matrix associated with cue C_h.

Other quantities of interest can be defined in terms of $P(A_1|S_1C_h)$ and $P(A_1|S_2C_h)$. Frequently we want to know the probability of an A_1 response independent of the stimulus event; namely,

$$P(A_1|C_h) = P(A_1|S_1C_h)\gamma_h + P(A_1|S_2C_h)(1-\gamma_h) .$$

Also of interest is the probability of a correct response (denoted c):

$$P(c|C_h) = P(A_1|S_1C_h)\gamma_h + P(A_2|S_2C_h)(1-\gamma_h) .$$

An incorrect response will be denoted $\bar{c}$.

Another important dependent variable is the subject's response time or latency. We will refer to his average latency, given response A_j, stimulus S_i, and cue C_h, as

$$E(L|A_jS_iC_h) ,$$

where L is the random variable representing the latency on a trial, and

E may be thought of as the expectation or averaging operator. Note that in contrast to the conditional response probabilities, the four latencies specified by $E(L|A_jS_iC_h)$ for each cue are independent. However, as is the case with the performance probabilities, we can define certain marginal quantities of interest in terms of $E(L|A_jS_iC_h)$. In fact,

$$E(L|S_iC_h) = E(L|A_1S_iC_h)P(A_1|S_iC_h) + E(L|A_2S_iC_h)P(A_2|S_iC_h)$$

is the average latency conditional on a C_h and S_i. Next,

$$E(L|A_jC_h) = E(L|S_1A_jC_h)\gamma_h + E(L|S_2A_jC_h)(1-\gamma_h)$$

is the average latency conditional on an A_j response and C_h. Also of interest are

$$E(L|cC_h) = \frac{E(L|S_1A_1C_h)P(A_1|S_1C_h)\gamma_h + E(L|S_2A_2C_h)P(A_2|S_2C_h)(1-\gamma_h)}{P(A_1|S_1C_h)\gamma_h + P(A_2|S_2C_h)(1-\gamma_h)}$$

and

$$E(L|\bar{c}C_h) = \frac{E(L|S_1A_2C_h)P(A_2|S_1C_h)\gamma_h + E(L|S_2A_1C_h)P(A_1|S_2C_h)(1-\gamma_h)}{P(A_2|S_1C_h)\gamma_h + P(A_1|S_2C_h)(1-\gamma_h)} ,$$

which are the expressions for the average latency given a correct and incorrect response respectively for cue C_h. Finally,

$$E(L|C_h) = E(L|S_1C_h)\gamma_h + E(L|S_2C_h)(1-\gamma_h)$$

is the overall latency for cue C_h.

DEVELOPMENT OF THE RECOGNITION-CONFUSION MODEL

In this section, a model for the experimental situation is presented and some predictions are derived. Two cases of the general model that seem germane to the present experiment are examined, and several aspects of the subject's performance are derived. Finally, a few comments will be made concerning the relationship of the model suggested here, to other current formulations.

Throughout the theoretical section we shall drop the subscripts and superscripts referring to cue lights, since these are important only in defining the result of a manipulation of the bias parameter in the models considered here. Theoretically, all the points generated by manipulation of the subject's bias should lie on the same ROC (receiver-operating characteristic) curve.

We will now state the axioms of the general model in a verbal fashion and then explicate them through the use of matrices.

Axioms:

1. At the instant of the stimulus offset, a random sample, S, of the symbols in the display is registered by the subject. The sample is of fixed size d.

2a. The subject processes, or scans, these symbols sequentially in a random order. Each symbol scanned is relegated to one of three classes, the assignment of which is represented by the following hypothetical state:

   i) s_0, the state corresponding to an assignment of the scanned symbol to the class of noise symbols,

ii) s_1, the state corresponding to an S_1 assignment, and

iii) s_2, the state corresponding to an S_2 assignment.

It is assumed that the processing time for each symbol is Δt, a constant over time and symbols.[1]

2b. The probability that s_i $(i = 0,1,2)$ is activated when Z_j $(j = 0,1,2)$ is processed is, in general, determined by: 1) when the symbol is processed relative to the other symbols in the sample S, 2) the particular class to which the symbol belongs.

3a. If s_1 or s_2 is activated, processing of the sample terminates and the subject makes the appropriate response (A_1 for s_1 and A_2 for s_2).

3b. If s_0 is activated, processing continues. We refer to this implicit "response" by the subject as A_0.

3c. If the subject processes all d of the sample symbols without activating either of the states s_1 or s_2, he guesses, responding A_1 with some fixed probability g. The quantity g will be referred to as the bias parameter. The value of this parameter will depend on the cue light that initiates the trial.

1/ The legitimacy of the assumption that the processing time is identical for all the symbols depends on the extent to which the subject uses the same number of observing or testing responses on each symbol. For example, a subject may be able to look for the joint occurrence of a circle and a horizontal or vertical bar in one Δt. If he "sees" either of these joint events, s_1 or s_2 is activated; otherwise s_0 is activated. In this case the Δt constancy over symbols would be justified.

However, it may be that the subject looks first for circularity; if he detects circularity, he looks to see if a horizontal or vertical bar lies within the circle. If circularity is absent, an s_0 is activated. If circularity is present and the next observing response leads to the detection of a vertical or horizontal bar, s_1 or s_2, respectively, is activated; otherwise s_0 is activated. In this instance, some Z_0 symbols would be processed faster than a Z_1 or Z_2 and the assumption would require modification.

We may represent these axioms by several matrices which explicate the properties of the axioms from the moment of stimulus presentation to the time of the subject's response.

First, we relate the stimulus display to the type of symbol processed at time $i\Delta t$, conditional on no s_1 or s_2 activation occurring prior to $i\Delta t$.

$$M = \begin{array}{c} \\ S_1 \\ S_2 \end{array} \begin{array}{c} \begin{array}{ccc} Z_0 & Z_1 & Z_2 \end{array} \\ \begin{bmatrix} \frac{D-1}{D} & \frac{1}{D} & 0 \\ \frac{D-1}{D} & 0 & \frac{1}{D} \end{bmatrix} \end{array},$$

where, as before, D is the number of symbols in the display. Note that M is not a function of i.

The matrix relating the type of symbol scanned at $i\Delta t$ to the identification or hypothetical activation state is

$$N_i = \begin{array}{c} \\ Z_0 \\ Z_1 \\ Z_2 \end{array} \begin{array}{c} \begin{array}{ccc} s_0 & s_1 & s_2 \end{array} \\ \begin{bmatrix} 1-x_i-y_i & x_i & y_i \\ 1-a_i-b_i & a_i & b_i \\ 1-a_i'-b_i' & b_i' & a_i' \end{bmatrix} \end{array},$$

where s_j denotes the hypothetical state corresponding to the assignment of a symbol to the s_j class. Note that N_i is undefined for the i^{th} step in the scan on a particular trial given that an s_1 or s_2 has already occurred, and its entries are functions of i.

The axioms further state that the identification state s_j is related to the subject's responses by the matrix:

$$Q = \begin{array}{c} \\ s_0 \\ s_1 \\ s_2 \end{array} \begin{array}{c} \begin{array}{ccc} A_0 & A_1 & A_2 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array}.$$

Since the axioms posit that the result of processing a symbol Z_i is always either an activation of state s_i (recognition) or one of the other two hypothetical states (confusion), we may refer to any model that satisfies the axioms as a recognition-confusion model.

ROC Curves

A characteristic of the subject's performance that is of considerable interest is his ROC curve. The abbreviation ROC signifies receiver operating characteristic; the ROC curve relates the quantity $P(A_1|S_1)$ to the quantity $P(A_1|S_2)$ when the stimulus conditions are fixed and learning variables are allowed to vary. In the present study, it is expected that manipulation of γ_h over the four cue lights will generate an ROC curve for each subject. The bias parameter associated with cue C_h should reflect the γ_h value.

In this section, we shall develop the ROC curve for the general case in a functional form and then explicitly for the model that is defined by letting the entries in the matrix N_i be constants.

An A_1 can follow presentation of S_1 in essentially three ways: by improper activation of s_1 by a Z_0; by proper activation of s_1 (i.e., activation of s_1 by Z_1); or by guessing. Further, the form of the function $P(A_1|S_1)$ will depend on whether or not the signal was included in the sample. Hence, we conditionalize on each of these events and then take the expectation with respect to these events; this yields the marginal $P(A_1|S_1)$.

Let $P(A_1|S_1,Z_1,i)$ be defined as the probability of an A_1 response, conditional upon the location of Z_1 at position i in the sample S. Then,

$$P(A_1|S_1,Z_1,i) = \sum_{j=1}^{i-1} x_j \cdot \prod_{k=0}^{j-1}(1-x_k-y_k) + \prod_{j=1}^{i-1}(1-x_j-y_j)a_i$$

$$+ \prod_{j=1}^{i-1}(1-x_j-y_j)(1-a_i-b_i)$$

$$\cdot \sum_{j'=i+1}^{d} x_{j'} \prod_{k=i+1}^{j'-1}(1-x_k\delta_{j'}-y_k\delta_{j'})$$

$$+ \prod_{j=1}^{i-1}(1-x_j-y_j)(1-a_i-b_i) \prod_{j'=i+1}^{d}(1-x_{j'}-y_{j'})g$$

$$= A_i + B_i a_i + C_i(1-a_i-b_i) + D_i(1-a_i-b_i)g \ ,$$

where $\delta_{j'} = \begin{cases} 0 & \text{if } j' = i+1 \\ 1 & \text{otherwise,} \end{cases}$

and $x_0 = y_0 = 0$.

Similarly,

$$P(A_1|S_2,Z_2,i) = \sum_{j=1}^{i-1} x_j \prod_{k=0}^{j-1}(1-x_k-y_k) + \prod_{j=1}^{i-1}(1-x_j-y_j)b'_i$$

$$+ \prod_{j=1}^{i-1}(1-x_j-y_j)(1-a'_i-b'_i)$$

$$\cdot \sum_{j'=i+1}^{d} x_{j'} \prod_{k=i+1}^{j'-1}(1-x_k\delta_{j'}-y_k\delta_{j'})$$

$$+ \prod_{j=1}^{i-1}(1-x_j-y_j)(1-a'_i-b'_i) \prod_{j'=i+1}^{d}(1-x_{j'}-y_{j'})g$$

$$= A_i + B_i b'_i + C_i(1-a'_i-b'_i) + D_i(1-a'_i-b'_i)g \ .$$

If the signal is not in the sample, and if we let $P(A_1|S_1,Z_1 \notin S)$ be the probability of an A_1 response conditional on Z_1 not being in the sample, then

$$P(A_1|S_1, Z_1 \notin S) = \sum_{j=1}^{d} x_j \prod_{k=0}^{j-1} (1-x_k-y_k) + \prod_{j'=1}^{d} (1-x_{j'}-y_{j'})g \ .$$

The corresponding expression for $P(A_1|S_2, Z_2 \notin S)$ is the same.

Next,

$$P(A_1|S_1) = E_i\{P(A_1|S_1, Z_1, i)\} + P(A_1, Z_1 \notin S|S_1) \ ,$$

$$P(A_1|S_2) = E_i\{P(A_1|S_2, Z_2, i)\} + P(A_1, Z_2 \notin S|S_2) \ ,$$

where E_i denotes the expectation with respect to i, the position on which the signal happens to fall within S and $P(A_1, Z_1 \notin S|S_1) = (1 - \frac{d}{D})P(A_1|S_1, Z_1 \notin S)$. Thus, $P(A_1|S_1) = E_i\{A_i + B_i a_i + C_i(1-a_i-b_i) + D_i(1-a_i-b_i)g\} + E + Fg$, where $E = (1 - \frac{d}{D}) \sum_{j=1}^{d} x_j \prod_{k=0}^{j-1} (1-x_k-y_k)$ and $F = (1 - \frac{d}{D}) \prod_{j'=1}^{d} (1-x_{j'}-y_{j'})$. A similar expression holds for $P(A_1|S_2)$.

Solving the expressions for $P(A_1|S_1)$ and $P(A_1|S_2)$ for g and setting the resultant formulae equal to one another, we can write $P(A_1|S_1)$ as a function of $P(A_1|S_2)$:

$$P(A_1|S_1) = \left[\frac{E_i[D_i(1-a_i-b_i)] + F}{E_i[D_i(1-a_i'-b_i')] + F}\right] \{P(A_1|S_2) - E_i[A_i + B_i b_i' + C_i(1-a_i'-b_i')] - E\}$$

$$+ E_i[A_i + B_i a_i + C_i(1-a_i-b_i)] + E \ .$$

Since the coefficients are independent of g, it follows that $P(A_1|S_1)$ is a linear function of $P(A_1|S_2)$ as the bias parameter g varies. That is, the ROC curve is a straight line whose slope is $\dfrac{E_i[D_i(1-a_i-b_i)] + F}{E_i[D_i(1-a_i'-b_i')] + F}$.

For this model, a necessary and sufficient condition for the line to be of slope 1 is that $E_i[D_i(1-a_i-b_i)] = E_i[D_i(1-a'_i-b'_i)]$. The y intercept is given by

$$E_i[A_i + B_i a_i + C_i(1-a_i-b_i)] + E$$

$$- \frac{E_i[D_i(1-a_i-b_i)]}{E_i[D_i(1-a'_i-b'_i)]} \{E_i[A_i + B_i b'_i + C_i(1-a'_i-b'_i)] + E\} .$$

The linearity of the ROC curve follows from the fact that both $P(A_1|S_1)$ and $P(A_2|S_2)$ are linear functions of g. The intuition behind the slope,

$$\frac{E_i[D_i(1-a_i-b_i)] + F}{E_i[D_i(1-a'_i-b'_i)] + F} ,$$

is that if $a_i + b_i$ is, on the average, small relative to $a'_i + b'_i$, the subject uses his guessing bias more on S_1 trials than on S_2 trials; hence, if g increases under these conditions, $P(A_1|S_1)$ changes more than does $P(A_1|S_2)$ and the slope is greater than one. If $a_i + b_i$ is large relative to $a'_i + b'_i$, the reverse holds.

On the other hand, the effect of processing a Z_0 is the same on the average for S_1 and S_2 trials and therefore does not affect the slope.

When the activation process is assumed to be invariant over time,

$$N_i = N = \begin{array}{c} \\ Z_0 \\ Z_1 \\ Z_2 \end{array} \begin{array}{c} \begin{array}{ccc} s_0 & s_1 & s_2 \end{array} \\ \begin{bmatrix} 1-x-y & x & y \\ 1-a-b & a & b \\ 1-a'-b' & b' & a' \end{bmatrix} \end{array} .$$

The ROC curve can again be developed by first deriving

$$E_i[P(A_1|S_1,Z_1,i)] = \frac{1}{D}\left\{\frac{x}{x+y}\left[d - \frac{1-(1-x-y)^d}{x+y}\right] + a\,\frac{1-(1-x-y)^d}{x+y}\right.$$

$$\left. + (1-a-b)x\left[\frac{1-(1-x-y)^d}{(x+y)^2} - \frac{d(1-x-y)^{d-1}}{x+y}\right] + d(1-a-b)g(1-x-y)^{d-1}\right\} .$$

It should now be evident that $E_i[P(A_1|S_2,Z_2 i)]$ is of exactly the same form except that a' and b' are substituted for a and b respectively. The terms $P(A_1,Z_1 \notin S|S_1)$ and $P(A_1,Z_2 \notin S|S_1)$ are developed as before and combined with $E_i[P(A_1|S_1,Z_1,i)]$ and $E_i[P(A_1|S_2,Z_2,i)]$ respectively to obtain $P(A_1|S_1)$ and $P(A_1|S_2)$. Solving for $P(A_1|S_1)$ in terms of $P(A_1|S_2)$ yields

$$P(A_1|S_1) = \frac{1-\frac{d}{D}(a+b) - (1-\frac{d}{D})(x+y)}{1-\frac{d}{D}(a'+b')-(1-\frac{d}{D})(x+y)}\left\{P(A_1|S_2) - \frac{1}{D}\left\{\frac{xd}{x+y} - \frac{x}{(x+y)^2}\right.\right.$$

$$\cdot [1-(1-x-y)^d] + \frac{b'}{x+y}[1-(1-x-y)^d] + (1-a'-b')\frac{x}{(x+y)}\left[\frac{1-(1-x-y)^d}{x+y}\right.$$

$$\left.\left.\left. - d(1-x-y)^{d-1}\right]\right\}\right\} + \frac{1}{D}\left\{\frac{xd}{x+y} - \frac{x}{(x+y)^2}[1-(1-x-y)^d]\right.$$

$$\left. + \frac{a}{x+y}[1-(1-x-y)^d] + (1-a-b)\frac{x}{x+y}\left[\frac{1-(1-x-y)^d}{x+y} - d(1-x-y)^{d-1}\right]\right\} .$$

Special Cases

We next consider two special cases of N_i that are of particular experimental interest:

$$\text{Case 1} \quad N_i = \begin{array}{c c} & \begin{array}{ccc} s_0 & s_1 & s_2 \end{array} \\ \begin{array}{c} Z_0 \\ Z_1 \\ Z_2 \end{array} & \begin{bmatrix} u & (1-u)q & (1-u)(1-q) \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

$$\text{Case 2} \quad N_i = \begin{array}{c c} & \begin{array}{ccc} s_0 & s_1 & s_2 \end{array} \\ \begin{array}{c} Z_0 \\ Z_1 \\ Z_2 \end{array} & \begin{bmatrix} v^i & q(1-v)^i & (1-v^i)(1-q) \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

We see that the basic structure of these cases is the same, including the number of parameters; the difference is that one is a function of i and the other is constant. An important property common to both is that whenever the subject processes a signal symbol, he recognizes it with probability one. Case 1 implies a constant probability over time of an s_1 or s_2 activation by Z_0, but Case 2 implies that the likelihood of an improper activation increases over time. The first would hold if the subject is able somehow to recharge his "image" or trace of the sample S until all d symbols are processed. The second would apply if the members of the sample were fading geometrically so that, say, after time $i\Delta t$ the quantity v_i represented the proportion of the symbol remaining for the subject to process. It should be of interest to compare predictions for these two cases for some commonly measured dependent variables.

The expressions for each case for $P(A_1|S_1)$, $P(A_1|S_2)$, and the ROC function will be presented. These will be followed by development of four conditional latencies: $E(L|A_1S_1)$, $E(L|A_1S_2)$, $E(L|A_2S_1)$ and $E(L|A_2S_2)$. As indicated earlier, other quantities may be easily derived, once the expressions for these basic quantities are known. Following the procedure developed above, we can easily find $P(A_1|S_1)$, $P(A_1|S_2)$, and the ROC curve for Case 1.

$$P(A_1|S_1) = \frac{1}{2} + \frac{1}{2D}\frac{1-u^d}{1-u} + (1-\frac{d}{D})u^d(g-\frac{1}{2}) \ ,$$

$$P(A_1|S_2) = \frac{1}{2} - \frac{1}{2D}\frac{1-u^d}{1-u} + (1-\frac{d}{D})u^d(g-\frac{1}{2}) \ ,$$

$$P(A_1|S_1) = P(A_1|S_2) + \frac{1}{D}\frac{1-u^d}{1-u} \ .$$

Similarly, for Case 2:

$$P(A_1|S_1) = \frac{1}{2} + \frac{1}{2D}\sum_{i=1}^{d} v^{\frac{i(i-1)}{2}} + (1-\frac{d}{D})v^{\frac{d(d+1)}{2}}(g-\frac{1}{2}) \ ,$$

$$P(A_1|S_2) = \frac{1}{2} - \frac{1}{2D}\sum_{i=1}^{d} v^{\frac{i(i-1)}{2}} + (1-\frac{d}{D})v^{\frac{d(d+1)}{2}}(g-\frac{1}{2}) \ ,$$

$$P(A_1|S_1) = P(A_1|S_2) + \frac{1}{D}\sum_{i=1}^{d} v^{\frac{i(i-1)}{2}} \ .$$

Let us begin the latency derivations by assuming that $L = (\delta\tau + \ell)\Delta t + t_0$ where L is the total latency or response time, τ is a variable which represents the time in Δt units to arrive at a guessing decision, $\delta = \begin{cases} 1 & \text{if guessing occurs,} \\ 0 & \text{otherwise,} \end{cases}$ ℓ is the random component representing the processing time in Δt units, and t_0 is the duration contributed by the subject's motor response. When g is noticeably greater than $\frac{1}{2}$, it may be that the associated guessing time is shorter for A_1 than for A_2, and when g is less than $\frac{1}{2}$, the guessing time for A_1 may be longer than that for A_2 (see Friedman, Burke, Cole, Keller, Millward, and Estes, 1964). In order to take account of this possible difference in the guessing latency component due to a difference in guessing bias, we suppose that two guessing latencies exist, one for the preferred response (bias parameter greater than $\frac{1}{2}$) and one for the non-preferred response (bias parameter less than $\frac{1}{2}$). We will distinguish the two latency components by appending subscripts to τ. When we are comparing theoretical predictions for latencies conditionalized on A_1 and A_2 responses, we shall append the subscripts 1 and 2, respectively, to allow for the possibility that when $g \neq \frac{1}{2}$, $\tau_1 \neq \tau_2$. In general, the preferred response guessing latency will be denoted τ_p and that of the non-preferred guessing latency will be denoted $\tau_{p'}$.

For our purposes, the time required to process one symbol (Δt) and the motor response time (t_0) can be considered as constants to be estimated from the data. Hence, our latency results will be derived in terms of ℓ and τ.

For Case 1, the conditional latencies of interest can be written

$$E(\ell+\delta\tau|A_1S_1) = \frac{\frac{1-u^d}{2(1-u)}[1+\frac{1}{D}] + (1-\frac{d}{D})[du^d(g-\frac{1}{2}) + gu^d\tau]}{\frac{1}{2} + \frac{1}{2D}\frac{1-u^d}{1-u} + (1-\frac{d}{D})u^d(g-\frac{1}{2})},$$

$$E(\ell+\delta\tau|A_1S_2) = \frac{\frac{-(1+u)(1-u^d)}{2D(1-u)^2} + \frac{d(1+u^d)}{2D(1-u)} + (1-\frac{d}{D})\{\frac{1}{2}\frac{(1-u^d)}{1-u} + du^d(g-\frac{1}{2}) + gu^d\tau\}}{\frac{1}{2} - \frac{1}{2D}\frac{1-u^d}{1-u} + (1-\frac{d}{D})u^d(g-\frac{1}{2})},$$

$$E(\ell+\delta\tau|A_2S_2) = \frac{\frac{1-u^d}{2(1-u)}(1+\frac{1}{D}) + (1-\frac{d}{D})[du^d(\frac{1}{2}-g) + (1-g)u^d\tau]}{\frac{1}{2} + \frac{1}{2D}\frac{1-u^d}{1-u} + (1-\frac{d}{D})u^d(\frac{1}{2}-g)},$$

$$E(\ell+\delta\tau|A_2S_1) = \frac{\frac{-(1+u)(1-u^d)}{2D(1-u)^2} + \frac{d(1+u^d)}{2D(1-u)} + (1-\frac{d}{D})\{\frac{1}{2}\frac{1-u^d}{1-u} + du^d(\frac{1}{2}-g) + (1-g)u^d\tau\}}{\frac{1}{2} - \frac{1}{2D}\frac{1-u^d}{1-u} + (1-\frac{d}{D})u^d(\frac{1}{2}-g)}.$$

The latency expressions for Case 2 are similar in structure to their parallels in Case 1.

$$E(\ell+\delta\tau|A_1S_1)$$

$$= \frac{\frac{1-v^d}{2(1-v)}[1-\frac{d}{D}-\frac{v}{(1-v)D}]+\frac{d}{2D(1-v)}+\frac{1}{2D}\sum_{i=1}^{d} i\, v^{\frac{i(i-1)}{2}} +(1-\frac{d}{D})[d(g-\frac{1}{2})+g\tau]\, v^{\frac{d(d+1)}{2}}}{\frac{1}{2}+\frac{1}{2D}\sum_{i=1}^{d} v^{\frac{i(i-1)}{2}} + (1-\frac{d}{D})v^{\frac{d(d+1)}{2}}(g-\frac{1}{2})},$$

$E(\ell+\delta\tau|A_1S_2)$

$$= \frac{\frac{1-v^d}{2(1-v)}[1-\frac{d}{D}-\frac{v}{(1-v)D}]+\frac{d}{2D(1-v)}-\frac{1}{2D}\sum_{i=1}^{d} i v^{\frac{i(i-1)}{2}}+(1-\frac{d}{D})[d(g-\frac{1}{2})+g\tau]v^{\frac{d(d+1)}{2}}}{\frac{1}{2}-\frac{1}{2D}\sum_{i=1}^{d} v^{\frac{i(i-1)}{2}}+(1-\frac{d}{D})v^{\frac{d(d+1)}{2}}(g-\frac{1}{2})},$$

$E(\ell+\delta\tau|A_2S_2)$

$$= \frac{\frac{1-v^d}{2(1-v)}[1-\frac{d}{D}-\frac{v}{(1-v)D}]+\frac{d}{2D(1-v)}+\frac{1}{2D}\sum_{i=1}^{d} i v^{\frac{i(i-1)}{2}}+(1-\frac{d}{D})[d(\frac{1}{2}-g)+(1-g)\tau]v^{\frac{d(d+1)}{2}}}{\frac{1}{2}+\frac{1}{2D}\sum_{i=1}^{d} v^{\frac{i(i-1)}{2}}+(1-\frac{d}{D})v^{\frac{d(d+1)}{2}}(\frac{1}{2}-g)},$$

$E(\ell+\delta\tau|A_2S_1)$

$$= \frac{\frac{1-v^d}{2(1-v)}[1-\frac{d}{D}-\frac{v}{(1-v)D}]+\frac{d}{2D(1-v)}-\frac{1}{2D}\sum_{i=1}^{d} i v^{\frac{i(i-1)}{2}}+(1-\frac{d}{D})[d(\frac{1}{2}-g)+(1-g)\tau]v^{\frac{d(d+1)}{2}}}{\frac{1}{2}-\frac{1}{2D}\sum_{i=1}^{d} v^{\frac{i(i-1)}{2}}+(1-\frac{d}{D})v^{\frac{d(d+1)}{2}}(\frac{1}{2}-g)}.$$

It is appropriate to mention a few properties common to both models. For convenience, these properties will be presented in terms of the parameters of Case 1.

For $u = 1$ and $d < D$, the latency $E(\ell+\delta\tau|A_1S_1)$ is an increasing function of the bias parameter g, but $E(\ell+\delta\tau|A_2S_2)$ is a decreasing function of g. The quantities $E(\ell+\delta\tau|A_1S_2)$ and $E(\ell+\delta\tau|A_2S_1)$ are independent of g when $u = 1$. When $0 < u < 1$, $d < D$, it is expected that the latencies conditional on an A_1 response are increasing functions of g since guesses always occur after all d of the sampled symbols are processed and hence are associated with the longest latencies. Increasing g augments the proportion of A_1 latencies that are associated with guessing responses; it causes a decrement in the proportion of A_2 latencies that are associated with a guessing response. The theorem that A_1 latencies are increasing functions of g but A_2 latencies are decreasing functions of g for all u when $d < D$, has not been proved. However, calculation of the conditional latencies on a computer for various values of the parameters has shown this to be true.

To compare the relations among the conditional latencies, subscripts are appended to τ which serve to indicate the associated guessing response. It will be assumed for simplicity that $d \geq 1$.

When $u = 0$, all four of the latencies reduce to

$$E(\ell + \delta\tau_k | A_i S_j) = 1 ,$$

but when $u = 1$,

$$E(\ell + \delta\tau_1 | A_1 S_2) = d + \tau_1, \; E(\ell + \delta\tau_2 | A_2 S_1) = d + \tau_2 ,$$

$$E(\ell + \delta\tau_1 | A_1 S_1) = \frac{\frac{d(d+1)}{2D} + (1-\frac{d}{D})g[d+\tau_1]}{\frac{d}{D} + (1-\frac{d}{D})g} ,$$

and

$$E(\ell + \delta\tau_2 | A_2 S_2) = \frac{\frac{d(d+1)}{2D} + (1-\frac{d}{D})(1-g)[d+\tau_2]}{\frac{d}{D} + (1-\frac{d}{D})(1-g)} .$$

It can be shown in this case $(u = 1)$ that

$$E(\ell + \delta\tau_1 | A_1S_2) \geq E(\ell + \delta\tau_1 | A_1S_1) ,$$

$$E(\ell + \delta\tau_2 | A_2S_1) \geq E(\ell + \delta\tau_2 | A_2S_2) ,$$

and equality holds among all four when $d = 1$, $\tau_k = 0$ $(k = 1,2)$. Also, it can be seen that under the condition $u = 1$, the τ_k magnitudes as well as g determine the ordering of two latencies conditionalized on different responses. If $\tau_1 = \tau_2$ and $u = 1$, then

$$E(\ell + \delta\tau | A_1S_2) = E(\ell + \delta\tau | A_2S_1) ,$$

and

$$E(\ell + \delta\tau | A_1S_1) = E(\ell + \delta\tau | A_2S_2) .$$

Note that an increase in g may be expected to lead to an increase in $E(\ell + \delta\tau | A_1S_1)$ and $E(\ell + \delta\tau | A_1S_2)$ unless g goes from less than $\frac{1}{2}$ to greater than $\frac{1}{2}$; in this case if $\tau_p > \tau_{np}$, the direction of change will depend on the relative magnitudes of $\tau_p, \tau_{p'}$ and g.

When $d = D > 1$ $(0 < u < 1)$,

$$E(\ell + \delta\tau | A_1S_1) = E(\ell + \delta\tau | A_2S_2) < E(\ell + \delta\tau | A_1S_2) = E(\ell + \delta\tau | A_2S_1) .$$

Note that in this case the incorrect latencies are shorter than the correct latencies.

When $d = 1$, all the conditional latencies $E(\ell + \delta\tau | A_1S_1)$, $E(\ell + \delta\tau | A_1S_2)$, $E(\ell + \delta\tau | A_2S_2)$, $E(\ell + \delta\tau | A_2S_1)$ assume the value 1.

Recognition-Confusion Model in Relation to Other Current Models

One way to exhibit similarities among models of behavior is to consider a more general model and examine the conditions under which the general model reduces to the various special cases. In this section we shall use this method to bring out resemblances among the present model, the serial processing model (Estes and Taylor, 1964, or Estes and Taylor, 1965), and a finite-state detection model (Atkinson, 1963, or Atkinson and Kinchla, 1965).

It is necessary to modify two aspects of the recognition-confusion model in order to obtain the appropriate generalization. First, the activation matrix N_i is expanded to include an additional state called the uncertain state. Thus,

$$N_i' = \begin{array}{c} \\ z_0 \\ z_1 \\ z_2 \end{array} \begin{array}{c} \begin{array}{cccc} s_0 & s_1 & s_2 & s_3 \end{array} \\ \begin{bmatrix} a_{00} & a_{01} & a_{02} & a_{03} \\ a_{10} & a_{11} & a_{12} & a_{13} \\ a_{20} & a_{21} & a_{22} & a_{23} \end{bmatrix} \end{array},$$

where s_3 represents the uncertain state. The a_{jk} entry refers to the probability of activating state s_k, given that z_j is processed at time $i\Delta t$.

The result of activating s_3 at $i\Delta t$ is assumed to be a continuation of processing, A_0, unless the activation occurs at $d\Delta t$, in which case the subject is presumed to guess. Under these assumptions the effect of either an s_0 or s_3 activation is the same, and hence the matrix N_i in the recognition-confusion model presented earlier can be interpreted as a collapsed version of N_i'.

The second modification is that of allowing for the possibility that processing of the symbols can be brought to a halt due to a drop below threshold of all the symbol traces; when this occurs, we postulate that the subject responds by guessing.

The serial processing model (Estes and Taylor, 1964; Estes and Taylor, 1965) can now be obtained from the above formulation by stipulating that $d = D$,

$$N_1' = \begin{array}{c c} & \begin{array}{cccc} s_0 & s_1 & s_2 & s_3 \end{array} \\ \begin{array}{c} Z_0 \\ Z_1 \\ Z_2 \end{array} & \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array},$$

and that m symbols are processed with probability 1 (during the stimulus on-time) after which there exists a constant probability s on each succeeding Δt that the trace of the symbols will fall below threshold.

Although easily extended to more complex situations, the Atkinson detection model (1963) was designed basically for application to simple two-alternative forced-choice and yes-no signal detection experiments. Since the forced-choice model can be developed from the yes-no type of situation that occurs in each position or interval, we may examine the activation matrix as it would appear for the presentation of one of two stimuli:

$$N'' = \begin{array}{c c} & \begin{array}{ccc} s_1 & s_2 & s_3 \end{array} \\ \begin{array}{c} S_1 \\ S_2 \end{array} & \begin{bmatrix} a_{11} & a_{12} & 1-a_{11}-a_{12} \\ a_{21} & a_{22} & 1-a_{21}-a_{22} \end{bmatrix} \end{array}.$$

Previous experimentation had shown that $a_{12} = a_{21} = 0$ so that N'' was reduced to

$$N'' = \begin{array}{c} \\ S_1 \\ S_2 \end{array} \begin{array}{c} \begin{array}{ccc} s_1 & s_2 & s_3 \end{array} \\ \begin{bmatrix} \sigma_1 & 0 & 1-\sigma_1 \\ 0 & \sigma_2 & 1-\sigma_2 \end{bmatrix} \end{array} .$$

One way of obtaining N'' from more primitive considerations that are compatible with the present discussion is to assume that on each trial a sample S of stimulus elements (of size d_1) is drawn from the display. Then, using the established notation Z_j to refer to a stimulus element, N'' follows from the activation matrix:

$$N_i' = \begin{array}{c} \\ Z_0 \\ Z_1 \\ Z_2 \end{array} \begin{array}{c} \begin{array}{cccc} s_0 & s_1 & s_2 & s_3 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array} .$$

Under this interpretation, $\sigma_j = d_j/D$; that is, if the signal element is contained in the sample, the proper hypothetical state is activated, otherwise the subject is presumed to be in the uncertain state and therefore responds by using his guessing bias. Note that here stimulus conditions are assumed to be such that the trace remains above threshold until the sample is processed.

METHOD

Apparatus

The experimental apparatus employed was an automated two-field dual tachistoscope at the Institute for Mathematical Studies in the Social Sciences, Stanford University. The display terminus of the apparatus was located in a sound-proofed, air-conditioned room. It sat on a 30-in. high table and appeared as a wooden box 5 ft. 7 in. long, 4 ft. 1 in. wide, and 2 ft. 4 in. high, on four 8 in. legs. At each of the two ends was a subject station, which was formed by recessing a full-height 8 in. wide panel 10 in. into the box. In this panel was mounted a ground-glass rear-projection screen, 8 in. wide and 6 $\frac{3}{4}$ in. high, centered vertically. Behind the screen was a black metal plate bearing six lights and a large circular aperture, neither of which was visible unless illuminated. A plastic eyepiece was mounted flush with the outer face of the box, 10 in. in front of the screen, aligned in height with the circular aperture. Below each observation station was suspended a response panel, at lap height, 12 in. wide and 10 in. deep. This bore a vertical array of four rectangular buttons, each of which was 1 in. by $\frac{7}{8}$ in., and a horizontal array of two buttons of identical size.

Displays were projected onto the screen through the large aperture, providing an illuminated circle 2 $\frac{1}{16}$ in. in diameter. Stimuli were displayed in a random-access slide projector (Spindler & Sauppe model SLX-750) modified to mount a special light source (Sylvania electronic tube #R1131C) characterized by rise time within 0.05 msec. and decay time within 0.025 msec. A second projector, optically identical to the first but holding a single slide, served to illuminate the screen between stimulus exposures.

Both projectors were concealed within the display box.

On both response panels, the four buttons arrayed vertically on the left were used for confidence-rating choices; the possible ratings were 1, 2, 3, and 4, with 1 being the topmost, 2 the second from the top, down to 4 at the bottom. Of the two buttons arrayed horizontally, the left one was employed as the A_1 response for Station 1 and for A_2 at Station 2; and the right one was employed as the A_2 response at Station 1 and as the A_1 response at Station 2.

Of the six peripheral lights behind the screen, the two outer lights served to provide information feedback, E_1 or E_2. They appeared as yellow $\frac{1}{4}$ in. circles containing a black slash which was vertical for E_1, horizontal for E_2. The right/left position of E_1 and E_2 on the screen corresponded at each station to the right/left assignment of A_1 and A_2 on the response panel. The four inner lights appeared at both stations as $\frac{3}{16}$ in. circles that were colored, from left to right, blue-green, orange, light green, and red, roughly matched for apparent brightness. They were functionally ordered C_1, C_2, C_3, and C_4 across the screen, with C_1 on the side corresponding to A_1 and E_1.

Each subject station was equipped with an intercom unit connected with a master unit in the control room. The control system of the apparatus was located in an adjacent room, visually and acoustically isolated from the display room. The control system, developed by Iconix Incorporated, of Menlo Park, California, read in the program statement of a trial, set up the control functions, stepped through the cycle, and recorded subject information such as latency, response type, and confidence rating as well as the trial statement, and then stepped to the next program statement.

Programmed variables for the present experiment included the slide to be shown, the feedback alternative, and the cue lights to initiate the trials.

Stimulus program tapes were generated by a PDP-1 digital computer which randomized the trial sequences under the constraints mentioned in the procedure section. These program tapes were read into the control system by a Teletype BRPE high-speed mechanical reader (100 lines per second). The subject's response information, after being stored briefly in a buffer, was read out to a Teletype Model 33 unit, which yielded simultaneous print-out and punched paper tape. The print-out was employed primarily for calibrating the equipment during the two days of practice and for monitoring the subject's output on a day-to-day basis. Data reduction, on the other hand, was accomplished by transferring the paper tape information to magnetic tape, where it was accessible to processing by an IBM 7090.

Procedure

As mentioned earlier, each trial of the experiment was initiated by one of the four cues: C_1, C_2, C_3, or C_4. The four cues were colored lights arranged in a horizontal row. After the cue light had been on for 3 sec., a white pre-stimulus field appeared and remained on until the stimulus was displayed 2 sec. later. A warning click sounded over the intercom $\frac{1}{2}$ sec. before the stimulus display was exposed. The stimulus duration x was determined for each pair of subjects during a two-day practice period and was then held constant for the remainder of the experiment. When the stimulus field was turned off, a white post-stimulus field, identical to the pre-stimulus field in size and intensity was turned on; its duration (3 sec.) delimited the response interval for the subject. During this interval, he had to make an A_j response and then a response that reflected his confidence in his perception of the stimulus display. We will refer to a confidence rating k as CR_k ($k = 1,2,3,4$). The final 4 sec. of each trial contained the feedback to the subject indicating the proper response for that trial. The events of a complete trial and their temporal order are shown in Fig. 1.

The stimulus field consisted of an array of 15 upper-case consonants as Z_0 symbols and either Z_1 or Z_2, but never both. In terms of the notation developed earlier, $D = 16$. Sixteen such arrays were constructed by arranging a square matrix of 16 consonants under the constraint that every letter should appear exactly once in every position; otherwise the arrangement was random. Then, each S_1 array was constructed by replacing a consonant with Z_1, leaving the remaining noise symbols unchanged. The Z_1

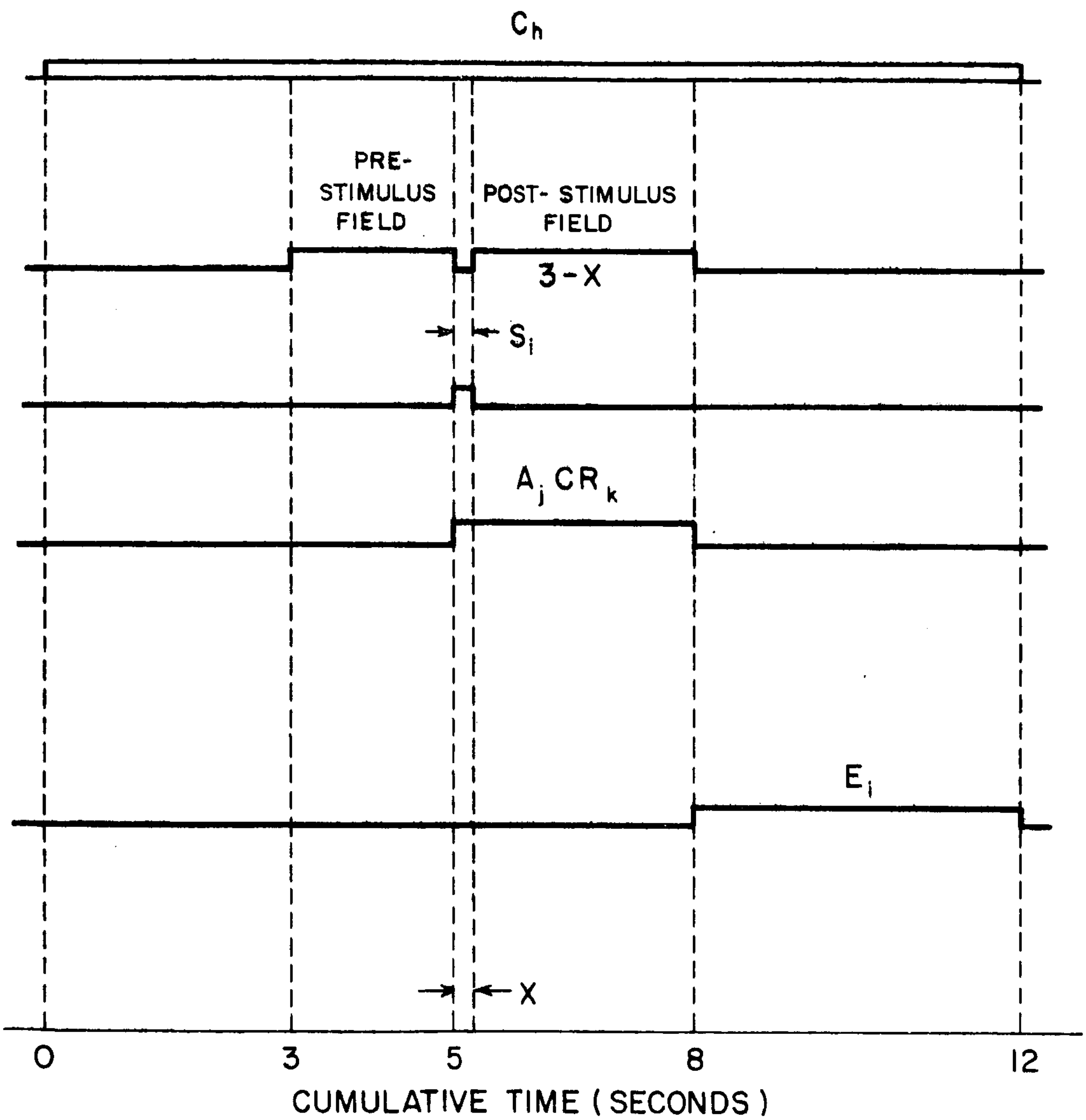

Figure 1. Trial Sequence.

was substituted for a differently positioned noise symbol in each array, resulting in sixteen S_1 stimuli. The S_2 arrays were formed in the same manner except that a Z_2 replaced the various consonants. Thus, the result was sixteen S_1 stimulus arrays and sixteen S_2 stimulus arrays. The symbol Φ was chosen as Z_1 and Θ was chosen as Z_2. The actual arrays, before introduction of the signal symbols, are shown in Table 1. Each array was reproduced on a glass slide for use in the random-access slide projector.

Twenty-four subjects, divided into twelve pairs of subjects, were used for the experiment. The subjects were run for two practice days for the purpose of adjusting the stimulus duration so that both subjects performed with less than perfect accuracy but at better than chance performance. The best a subject could do by chance for C_h was $P(c) = \gamma_h$ if $\gamma_h > \frac{1}{2}$ and $P(c) = 1 - \gamma_h$ if $\gamma_h < \frac{1}{2}$, and the worst was $P(c) = 1 - \gamma_h$ if $\gamma_h > \frac{1}{2}$ and $P(c) = \gamma_h$ if $\gamma_h < \frac{1}{2}$. The simple threshold model specified by the expressions:

$$P(A_1|S_1C_h) = \sigma_h + (1-\sigma_h)g_h \ ,$$

$$P(A_1|S_2C_h) = (1-\sigma_h)g_h \ ,$$

was used to estimate the subject's accuracy of perception (specified by σ_h) and his guessing bias (g_h). To ensure that, under this simple model, a subject was performing between chance and perfect accuracy, it was sufficient to manipulate the stimulus display duration so that $0 < \sigma_h < 1$; an estimate of σ_h was obtained from the expression $\sigma_h = P(A_1|S_1C_h) - P(A_1|S_2C_h)$.[2] Of course, no prespecified level of accuracy could be

2/ Note that when $1 - x_i - y_i = a_i = a_i' = 1$ in the recognition-confusion model, the simple threshold model outlined above is obtained with $\sigma_h = d/D$.

Table 1

Stimulus Arrays

$\underline{S}$ M W P	R $\underline{S}$ F G	K F $\underline{D}$ W	C Z M $\underline{H}$	Z W P B	X N Z D	F H X S	H X R N
F H K Z	B Z X P	G X C B	D F B G	$\underline{S}$ G D C	W $\underline{M}$ P R	R C $\underline{T}$ W	C K W $\underline{T}$
T C G D	C D M W	H R Z P	X S N T	N T R K	K G H C	D M P Z	B F S G
X R N B	T N H K	N S T M	K W R P	H F M X	F B S T	B K G N	M D P Z

W P T R	N T K Z	G K S C	T B C K	P D H T	M G B F	D R N M	B C G X
Z D N S	X P M D	N B R M	H S Z N	M N G K	P W H X	K T S F	T R F H
$\underline{G}$ K B F	R $\underline{H}$ C B	W P $\underline{T}$ X	F X W $\underline{M}$	S B F R	Z N K S	P Z X H	M W D N
C M X H	S G W F	Z H F D	R P D G	$\underline{W}$ X Z C	D $\underline{T}$ C R	G C $\underline{B}$ W	P Z K $\underline{S}$

Noise letters: B C D F G H K M N P R S T W X Z

S_1 stimuli: replaced underlined letters with ⦶.

S_2 stimuli: replaced underlined letters with ⊖.

obtained since the two subjects in a pair often were not equally sensitive to the type of stimulus displayed.

An additional reason for the two days' practice was to acquaint the subjects with the presentation frequencies (γ_h) and to familiarize them with Θ and Φ before beginning the test phase. This practice with the actual presentation frequencies, together with instructions that there existed a correlation of the cues with the presentation frequencies of the signals, was meant to minimize cue learning during the test phase. The test phase which followed required six additional days.

The four cues each occurred on one-fourth of the trials, and each cue was associated with a particular schedule of the two signals. One subject in each pair of subjects received the following cue-to-color assignment: C_1, red; C_2, green; C_3, yellow; and C_4, blue. The other subject received the assignment: C_1, blue; C_2, yellow; C_3, green; and C_4, red. During the test phase 240 trials were run per day with a constrained randomization for both cue and signal frequencies. After 160 trials the subjects were given a 1 min. rest break. Within an 80-trial block, each of the cues appeared 20 times; 15 of the C_1 trials were always S_1 trials $(\gamma_1 = .75)$, $\gamma_2 \times 20$ of the C_2 trials were S_1 trials, $\gamma_3 \times 20$ of the C_3 trials were S_1 trials, and five of the C_4 trials were always S_1 trials $(\gamma_4 = .25)$. An important characteristic of the experimental design was the symmetry in γ, that is, $\gamma_4 = 1 - \gamma_1$, and $\gamma_3 = 1 - \gamma_2$.

Thus, the subjects obtained the same number of S_1 and S_2 trials in an 80-trial block despite the cue-dependent schedule. For half the subjects, $\gamma_2 = .60$; we designated this set of subjects as Group 1. The other one-half of the subjects had a γ_2 value of $.90$; we designated this set of subjects as Group 2. Group 1 and Group 2 were each composed of twelve subjects. A tabular summary of the presentation schedule is presented in Table 2.

One-half of the subjects randomly selected in each group, were run on a schedule that included a completely new randomization for each day of practice and each day of the experimental phase. The other half of the subjects in each group were run on a random permutation of the first schedule. For this second set of subjects, the pool consisting of the randomizations for each of the eight days used for the first set was rearranged in a random fashion.

The subjects were run two at a time on the apparatus; each always had the same partner and the same station at the apparatus. The two subjects were placed at opposite ends of the tachistoscope, and neither could see the other or the experimenter, who was in the control room adjacent to the test room. Each subject had in front of him a panel equipped with buttons for his A_j response and four buttons which he used to indicate his confidence as to how clearly he saw the signal that he reported. While the experiment was in progress, the subject pressed his face to a viewing hood. The cue lights, stimulus display,

Table 2

Presentation Schedule

	C_h	γ_h	S_1 frequencies per 80-trial block	S_2 frequencies per 80-trial block
Group 1 12 subjects	C_1	.75	15	5
	C_2	.60	12	8
	C_3	.40	8	12
	C_4	.25	5	15
Group 2 12 subjects	C_1	.75	15	5
	C_2	.90	18	2
	C_3	.10	2	18
	C_4	.25	5	15

and feedback lights appeared at the proper time on a vertical screen which was grey until one of the lights was turned on behind it. The stimuli were presented with the same duration to both of the subjects in a subject-pair. The intensities of the tachistoscopic fields are given in Table 3. The ambient light level was unmeasurably low on the screens but provided 0.009 and 0.065 foot-candles illumination on the response panels at Station 1 and Station 2 respectively. The stimulus array, when displayed, subtended a visual angle of $5°10'$. A single symbol in the display subtended a visual angle of about $1°$. A vertical schematic of the physical arrangement of the apparatus and the subjects' positions with respect to it are shown in Fig. 2.

On day 1 of the experiment, each of the two subjects in a pair was arbitrarily assigned to one of the stations at the apparatus. A statement was read to the subjects concerning their obligations under the terms of the experiment and the conditions of remuneration. If both subjects agreed to these terms, they were given printed instructions which contained their actual tasks in the experiment. These instructions read as follows:

Table 3

Brightness of the Tachistoscopic Fields

(Measured in foot-candles)

	Station 1	Station 2
Pre-stimulus / Post-stimulus	4.43	1.51
Stimulus	2.45	1.80

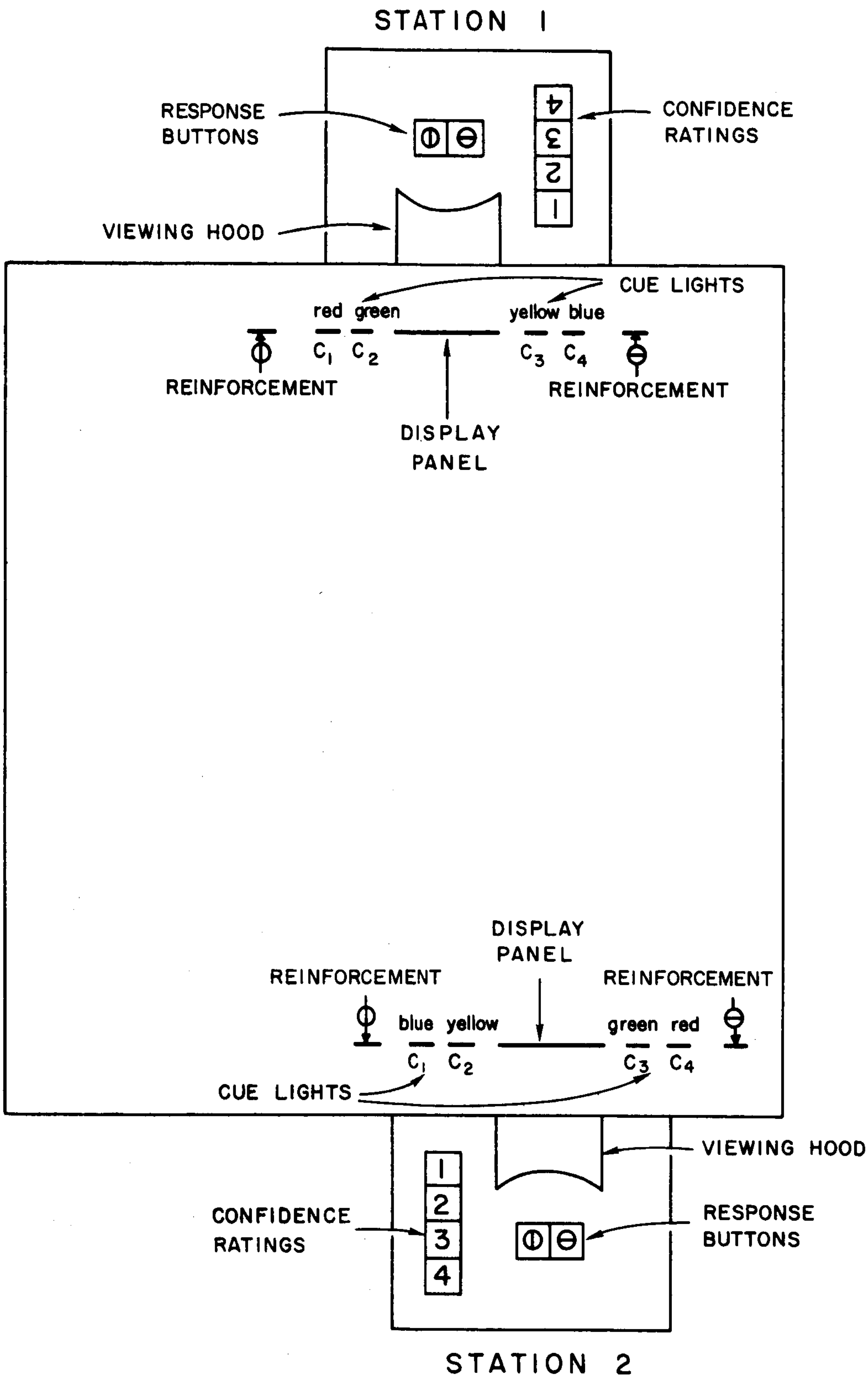

Figure 2. Display and Response Apparatus.

Instructions to Subjects

This is a device for presenting visual stimuli for brief intervals of time.

Periodically you will be presented a visual display. Each display will contain 15 letters and, in addition, either the symbol ⊖ or the symbol ⦶. Your task will be to ascertain to the best of your ability which of these two symbols is present on each trial, and to push the button corresponding to this choice.

It is not to be expected that you will see the symbol with perfect clarity every trial. To help you in your performance, four colored lights have been placed above the display panel. These lights are partially correlated with the frequency of appearance of the two symbols ⊖ and ⦶. On each trial, one of these colored lights will precede the presentation display. The symbol you see on the left response button is more likely to be preceded by one of the two colored lights on the left, while the symbol on the right response button is more likely to be preceded by one of the two colored lights on the right. Furthermore, of the two colored lights on the right, one is even more likely to precede the symbol on the right than the other; and of the two colored lights on the left, one is even more likely to precede the symbol on the left than the other. You may be able to improve your performance by using these colored lights along with your visual observation of the display.

At the beginning of every trial, the index finger of the right hand is to be placed over the right symbol response button and the index finger of the left hand should be over the left symbol response button. Make your symbol response with these fingers.

In addition to pushing the button of the symbol you think was contained in the display, you are to push one of the four buttons you see on the left; after making the other response. This will serve to show your degree of confidence in what you saw. If you are absolutely sure of what you saw, then push the "1" button; when you are relatively sure, push the "2" button; the "3" button when you are relatively unsure; and push the "4" button when you absolutely unsure, that is, guessing at random. Do not allow the confidence-rating process to interfere with the primary task of making a symbol Θ/Φ judgment; accomplish the latter first and then decide how sure you were of what you saw. Note that your confidence rating is to be based on your evaluation of your visual accuracy each trial, not your assurance of being correct. This is in spite of the fact that your primary responsibility with respect to the symbol response is to do as well as possible using both your visual impressions and the colored lights.

You have approximately three seconds after the display vanishes to make your responses; be sure to make a Θ/Φ response and a confidence-rating response on each trial. Immediately after you have made your response, a sign indicating which symbol appeared in the display will flash on; if Θ was presented, then a sign representing this symbol will come on, and similarly for a Φ symbol.

You can see an intercom phone on the apparatus. This is to be used to communicate with the experimenter if there appears to be a malfunction in the equipment; this should be used only in the event that something seems to be wrong in the trial sequence or presentation of the materials.

The intercom also serves the purpose of sounding a click just before the display is presented. Accompanying this click will be a bright disk on which you may fixate and on which the display will be presented.

Let us then reiterate a complete trial sequence:

1. Your right index finger is placed on the right symbol button and the left index finger is placed on the left symbol button.
2. A colored light will come on and stay on for the whole trial.
3. Shortly after this, the bright disk will come on.
4. Next, a click indicating the display is about to be presented will sound.
5. The symbol display will appear where the bright disk is, for a very short time.
6. Shortly after this, the bright disk will go off.
7. You will immediately make your decision as to which symbol you think appeared in the display and make the appropriate response with the left or right index finger. You need not wait for the bright disk to vanish. Get as many correct as you can, using what you saw and the colored lights.
8. You follow this with a rating of your confidence by pushing the confidence-rating button which corresponds to your evaluation of the accuracy with which you saw the symbol presented in the display.
9. Make the previous two responses on every trial, even if you have to guess.
10. A sign will flash on indicating which symbol actually was in the display for that trial.
11. End of trial. Beginning of next trial.

Please keep your face pressed comfortably against the hood except when you need to check the position of the response buttons when making your responses or during the brief rest period.

Any questions the subjects may have had after reading the instructions were answered by referring them to the appropriate section of the instructions. When the subjects were satisfied that they understood the experimental procedure, the ambient light level in the experimental room was lowered, the subjects were seated in comfortable, adjustable chairs at their respective stations, and the experiment began.

Each day, before beginning the session, the subjects were encouraged to refresh their memories by referring to the instructions if they felt uncertain about any aspect of the procedure.

Subjects

Group 1 and Group 2 each consisted of twelve subjects drawn from the Stanford University and Foothill College communities. All were students or wives of students between the ages of eighteen and thirty who were paid for their services. Visual acuity was required to be at least 20/20 after correction, but no subject had to be rejected on this criterion. English was the native language of all the subjects. Group assignment was on a random basis.

RESULTS

Cue Differentiation

The average proportion of A_1 responses given to each of the four cues for each day of the test phase is presented in Fig. 3; the results for Group 1 are given in Fig. 3a and for Group 2 in Fig. 3b. The ordering of $P(A_1|C_h)$ corresponds to the ordering of γ_h for both groups. Thus, manipulation of γ_h was associated with differences in the frequencies of A_1 responses.

We may present the results in Fig. 3 in a way that shows that the more γ_h deviates from $\frac{1}{2}$, the more extreme are the associated response frequencies. In Fig. 4 the result of averaging $P(A_1|C_1)$ and $P(A_2|C_4)$ can be compared with the result of averaging $P(A_1|C_2)$ and $P(A_2|C_3)$ for Groups 1 and 2. As is expected, in Group 1 where the outside cues (C_1 and C_4) are more highly correlated with S_1 and S_2 frequencies than are the middle cues (C_2 and C_3), the average of $P(A_1|C_1)$ and $P(A_2|C_4)$ is greater than the average of $P(A_1|C_2)$ and $P(A_2|C_3)$. In Group 2 the average of $P(A_1|C_1)$ and $P(A_2|C_4)$ is smaller than the average of $P(A_1|C_2)$ and $P(A_2|C_3)$, since for that group the middle cues were more highly correlated with the S_1 and S_2 presentations than were the outside cues.

Since γ_1 and $1 - \gamma_4$ were .75 for Groups 1 and 2, and γ_2 and γ_3 were different for the two groups, it is pertinent to ask if $P(A_1|C_1)$ and $P(A_1|C_4)$ are affected by these differences in γ_2 and γ_3. As noted earlier, in both groups γ_1 and γ_2 were greater than 1/2 and γ_3 and γ_4 were less than 1/2; furthermore, C_1 and C_2 were juxtaposed on one

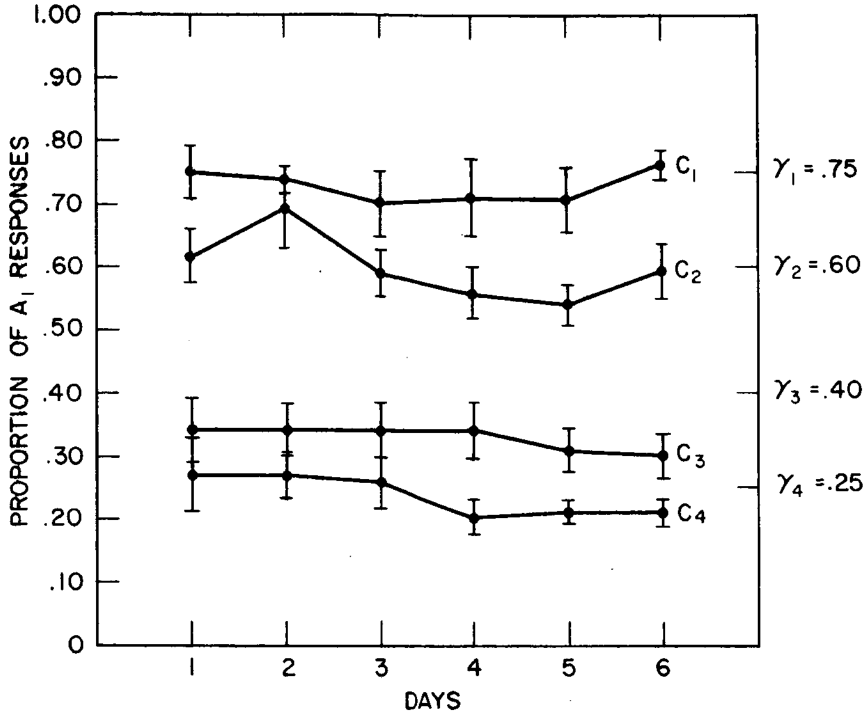

Figure 3a. $P(A_1|C_h)$ for Group 1.

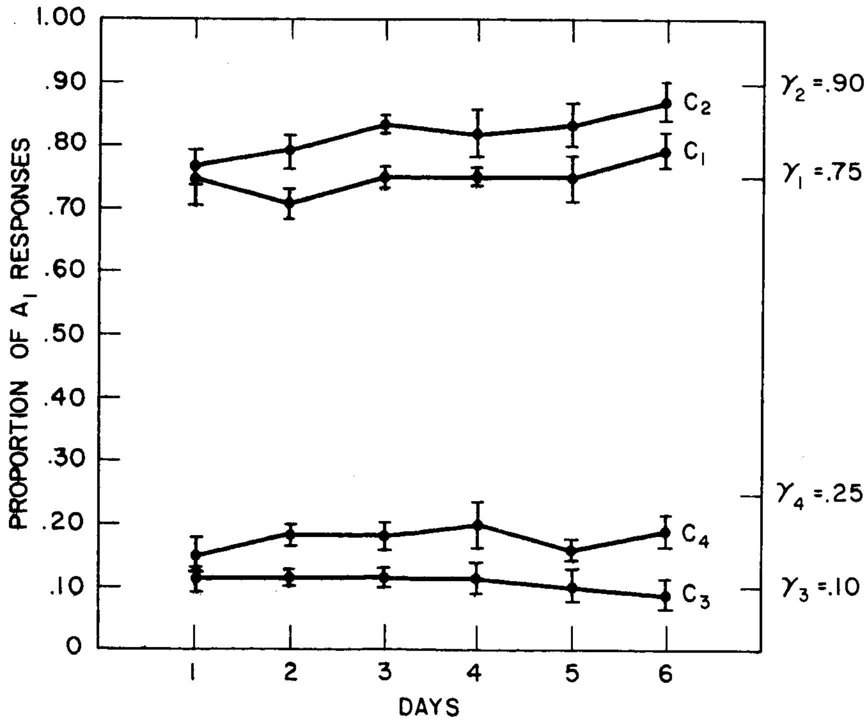

Figure 3b. $P(A_1|C_h)$ for Group 2.

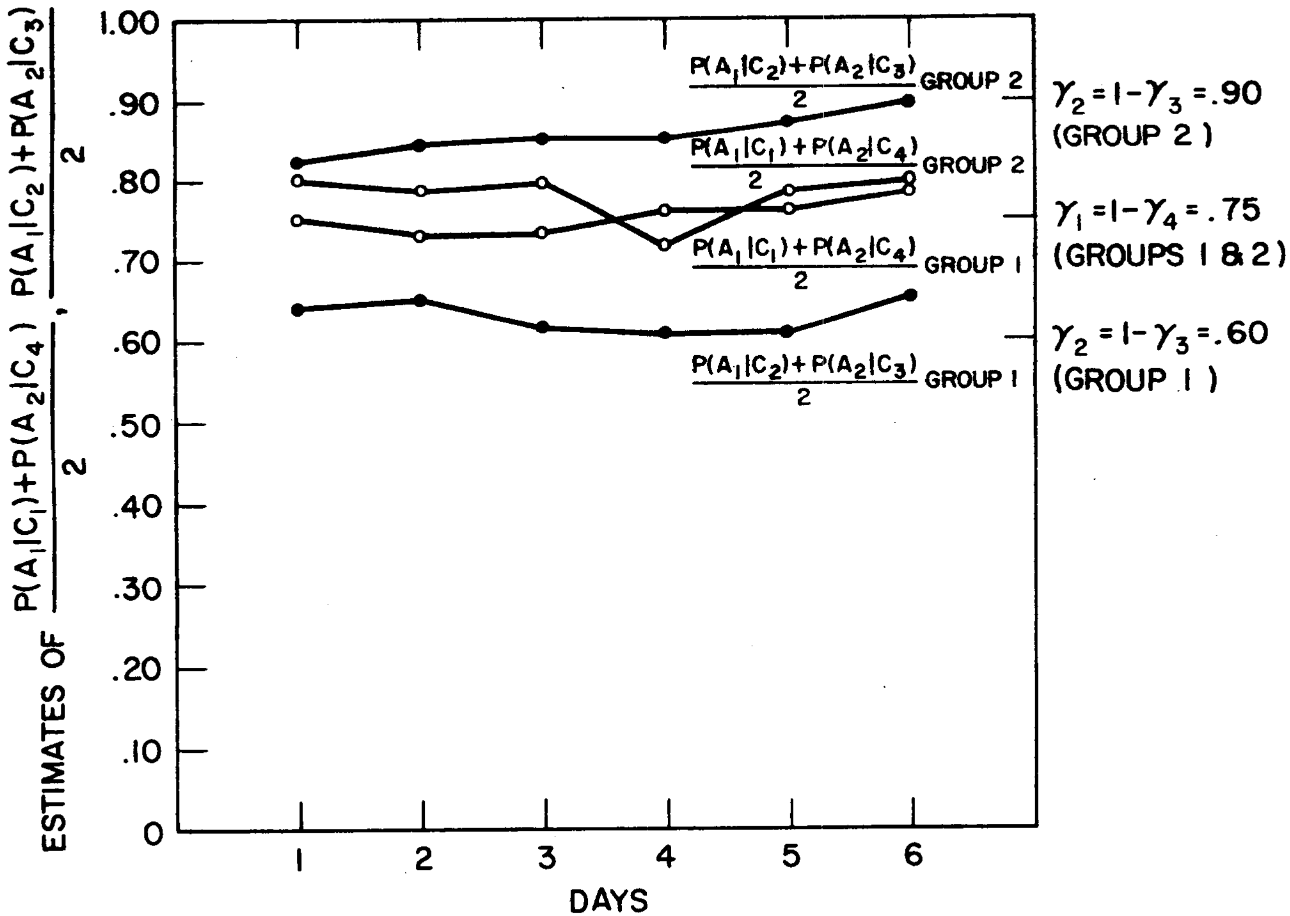

Figure 4. Proportion of A_1 Responses to High and Low Correlated Cues.

end of the array of cue lights, and C_3 and C_4 were juxtaposed on the other end. This arrangement should enhance generalization between C_1 and C_2 and between C_3 and C_4. If generalization was an important factor in the subjects' behavior, $P(A_1|C_1)$ should be larger in Group 2 than in Group 1, and $P(A_1|C_4)$ should be smaller in Group 2 than in Group 1. This prediction stems from the fact that $\gamma_2 > \gamma_1$, $\gamma_3 < \gamma_4$ for Group 2 and $\gamma_2 < \gamma_1$, $\gamma_3 > \gamma_4$ for Group 1. Table 4 presents observed values of $P(A_1|C_1)$ and $P(A_1|C_4)$ for both groups. A t-test was performed on $P(A_1|C_1)$ for the two groups, and a separate test was performed on $P(A_1|C_4)$. Then, the average of $P(A_1|C_1)$ and $P(A_2|C_4)$ for each subject was obtained and a t-test run on the difference in this quantity for the two groups. All of these tests led to an acceptance of the hypothesis that $P(A_1|C_1)$ and $P(A_1|C_4)$ were unaffected by manipulation of γ_2 and γ_3.

The overall $P(A_1)$ averaged over cues and subjects in each of the groups is shown for each test day in Fig. 5. A large deviation from $\frac{1}{2}$ would indicate a tendency for the subjects to make one of the responses more than the other, in spite of the symmetry of the schedule $[\gamma_1+\gamma_2+\gamma_3+\gamma_4)/4 = \frac{1}{2}]$. In terms of the models discussed earlier (Cases 1 and 2), a finding of this nature might be due to $\frac{1}{4}(g_1+g_2+g_3+g_4) \neq \frac{1}{2}$ or to $q \neq \frac{1}{2}$. There appears to be a small but fairly consistent tendency to respond A_2 more frequently than A_1. All but three subjects out of 24 made A_1 responses less than half of the time, and the marginal $P(A_1)$ averaged over days was .47 for both groups. This result does not seem to follow from an initial greater familiarity of the subjects with $\ominus$ since the two practice sessions should have brought equality in this respect.

Table 4.

Observed Values of $P(A_1|C_1)$ and $P(A_1|C_4)$

For Groups 1 and 2.

	$P(A_1\|C_1)$		$P(A_1\|C_4)$	
Subjects	Group 1	Group 2	Group 1	Group 2
1	.90	.82	.52	.16
2	.77	.77	.15	.17
3	.63	.93	.27	.10
4	.48	.65	.24	.22
5	.91	.81	.27	.15
6	.59	.71	.15	.26
7	.74	.73	.25	.23
8	.66	.76	.19	.19
9	.71	.70	.23	.10
10	.87	.76	.07	.21
11	.91	.82	.23	.18
12	.67	.65	.22	.19
Average	.74	.76	.22	.17

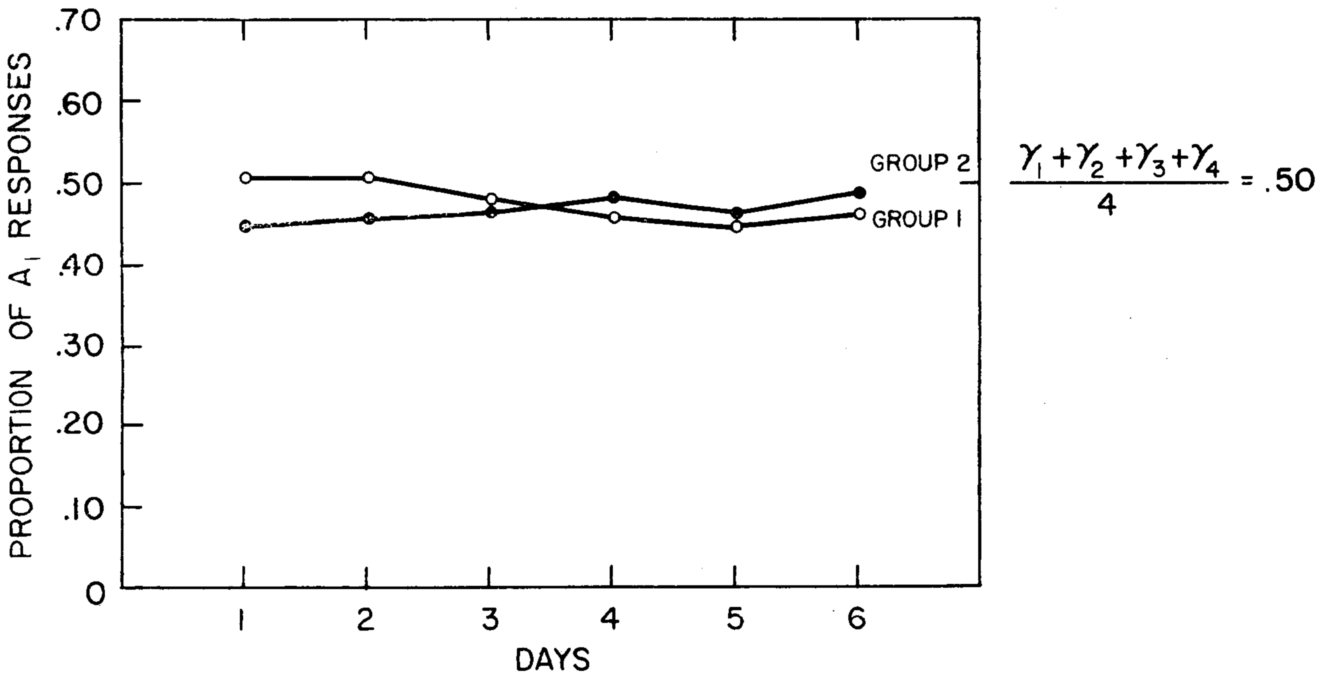

Figure 5. Average of $P(A_1)$ over Cues.

Sensitivity as Affected by Presentation Schedule

In the general model, when $a_i = a'_i = 1$, an index of a subject's sensitivity may be obtained by subtracting the proportion of incorrect A_1 responses from the proportion of correct A_1 responses $[P(A_1|S_1) - P(A_1|S_2)]$. This index (a function of d and $1 - x_i - y_i$) expresses the average likelihood that the subject processes the signal when it is contained in the sample of d symbols. We shall call this index σ.

The possibility has been suggested that with high cue-stimulus correlations, estimates of a subject's σ might be lower than estimates of his σ with lower cue-stimulus correlations (Atkinson, 1963). The reasoning is that since the subject can do quite well simply by appropriately biasing his responses when $|\gamma_h - \frac{1}{2}|$ is large, he may be induced to relax his attention on those trials initiated by a cue having a high correlation with S_1 or S_2.

Figure 6 shows daily estimates of the average σ for the relatively low correlated cues (σ_2 and σ_3 for Group 1, and σ_1 and σ_4 for Group 2) along with daily estimates of the average σ for the relatively high correlated cues. Call the average of σ_1 and σ_4, $\sigma_{1,4}$ and the average of σ_2 and σ_3, $\sigma_{2,3}$. Cues C_1 and C_4 were the high correlated cues in Group 1, but C_2 and C_3 were the high correlated cues in Group 2. A paired t-test showed that $\sigma_{2,3}$ was significantly different from $\sigma_{1,4}$ for Group 1 but not for Group 2. A paired t-test over both groups on the difference between σ for the highly correlated cues and σ for the low correlated cues was nonsignificant. On the other hand, there seems to be a suggestion in the data that σ is larger for the middle cues than for the outer cues since $\sigma_{2,3}$ tends to be greater than $\sigma_{1,4}$ for

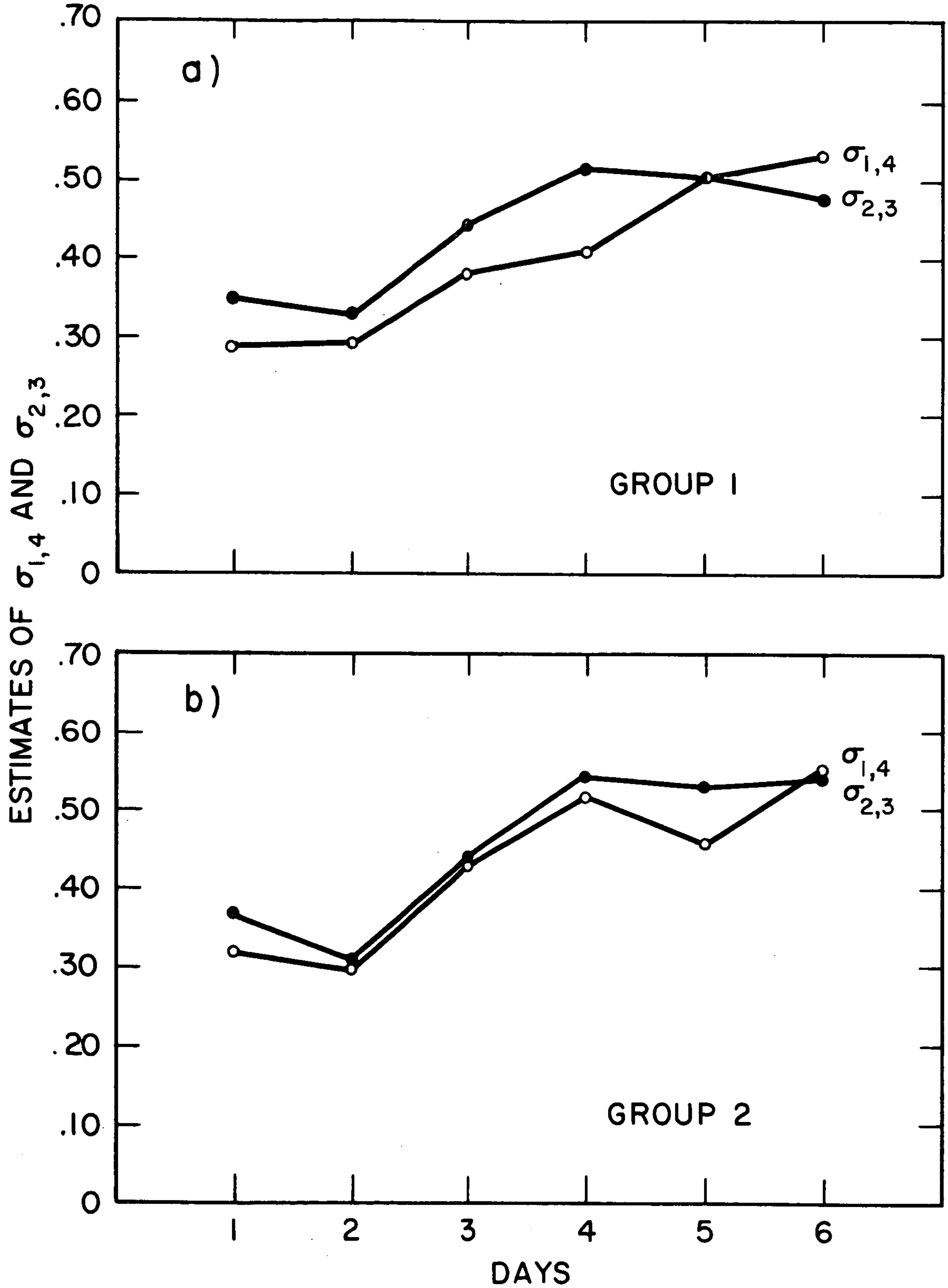

Figure 6. Sensitivity for High and Low Correlated Cues.

both groups. This problem will be treated further in the following section and in the discussion.

ROC Curves: One Parameter Fits

Previous work with a cued-detection task in an auditory setting has shown that the ROC curves for subjects performing in this type of task can be well described by straight lines of slope 1 (Kinchla, Townsend, Yellott, Atkinson, 1966). Recall that this situation obtains for the present model when $E_i(1 - a_i - b_i) = E_i(1 - a_i' - b_i')$.

The theory of signal detectability provides an alternative formulation which includes prediction of curvilinear ROC curves. Under this theory, the effect of an S_1 or S_2 presentation could be represented by a vector in k-space. Associated with S_1 and S_2 are two k-dimensional probability distributions. The subject behaves as if he knew the two distributions associated with the presentation of an S_1 or S_2, and employs these distributions to construct a likelihood ratio on each trial of the probability densities associated with the current vector. The subject is supposed to have established a cut-point on an axis of the logarithm of the likelihood ratios; when the log of the likelihood ratio which arises on a particular trial exceeds that cut-point, he makes a specified response, and if the log-likelihood ratio falls below the cut-point, he makes the other response.

The probability density functions of the log-likelihood ratios are usually assumed to be normal and the normalized distance between the means of the density functions is denoted d'. The quantities $P(A_1|S_1)$ and $P(A_1|S_2)$ may then be computed for any cut-point using cumulative normal curve tables and d'. For an account of signal detectability theory, see Swets, Tanner, and Birdsall (1961) or Green (1960).

In signal detectability theory, the analogue of the slope 1 assumption in the present model is the assumption that the variances of the S_1 and S_2 distributions are identical. Based on these assumptions, we can obtain one-parameter fits of both models to the individual ROC curves in the present experiment. The method used to fit the models was that of mean square orthogonal regression (see Cramér, 1946). This method seems more appropriate for fitting ROC functions than the usual regression technique since both axes represent dependent variables. For a straight line of slope 1, the method of orthogonal regression reduces to the ordinary least-squares fit. However, for the signal detectability analysis, the method of orthogonal regression prevents the artificial inflation of the error estimate given to points in the extremities of the ROC space which occurs with the usual regression technique.

The curvilinear fit is obtained by selecting the d' which minimizes the sum of the orthogonal distances (or deviations, the term we shall henceforth employ) of the observed points from the theoretical function. Similarly, we may fit the present model by varying the intercept until a least sum of the orthogonal deviations is obtained. The theoretical intercept, as noted earlier, is a function of d and the Z_0 confusion parameters. However, for present purposes, we can treat the intercept as a single parameter to be estimated by the above method. Thus, both models are fit by varying an index of the subject's sensitivity.

The results of these fits are plotted for each subject in Fig. 7 and Fig. 8. Table 5 presents the sum of the squared orthogonal deviations of the two models for each subject in Groups 1 and 2. The two models appear to do about equally well for Group 1, but the curvilinear fit seems

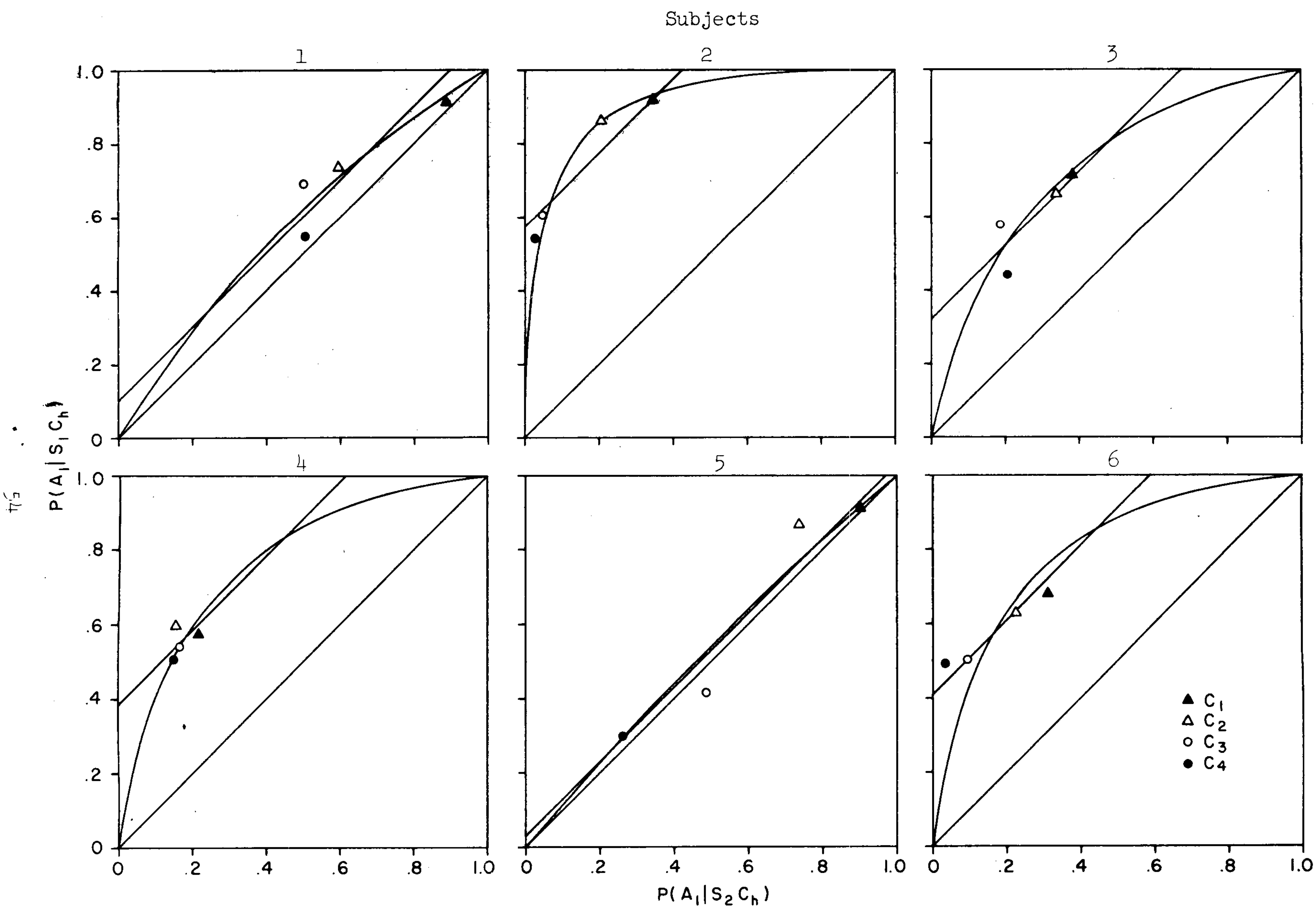

Figure 7a. Recognition-Confusion and Signal Detectability One-Parameter ROC Curves for Group 1.

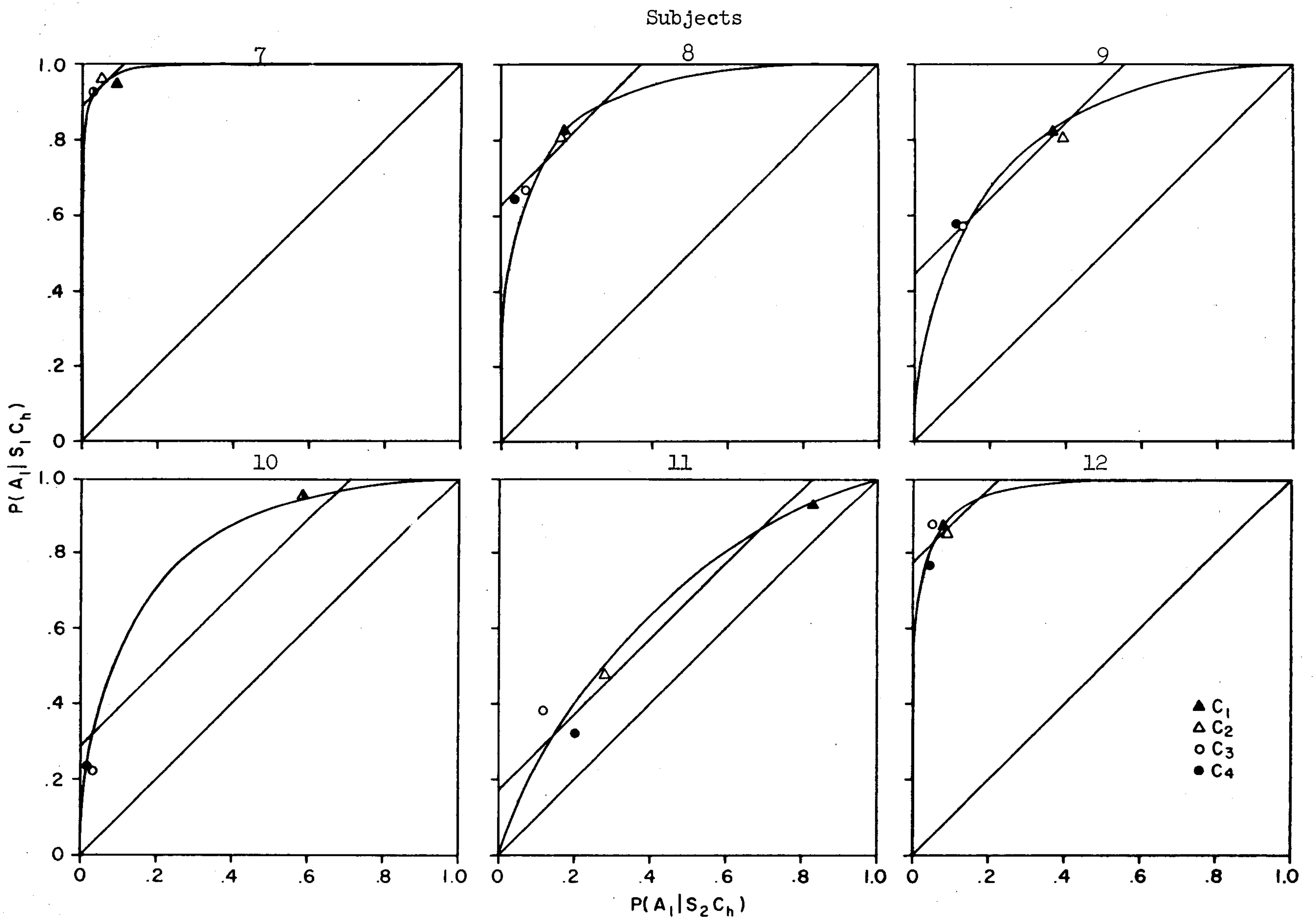

Figure 7b. Recognition-Confusion and Signal Detectability One-Parameter ROC Curves for Group 1.

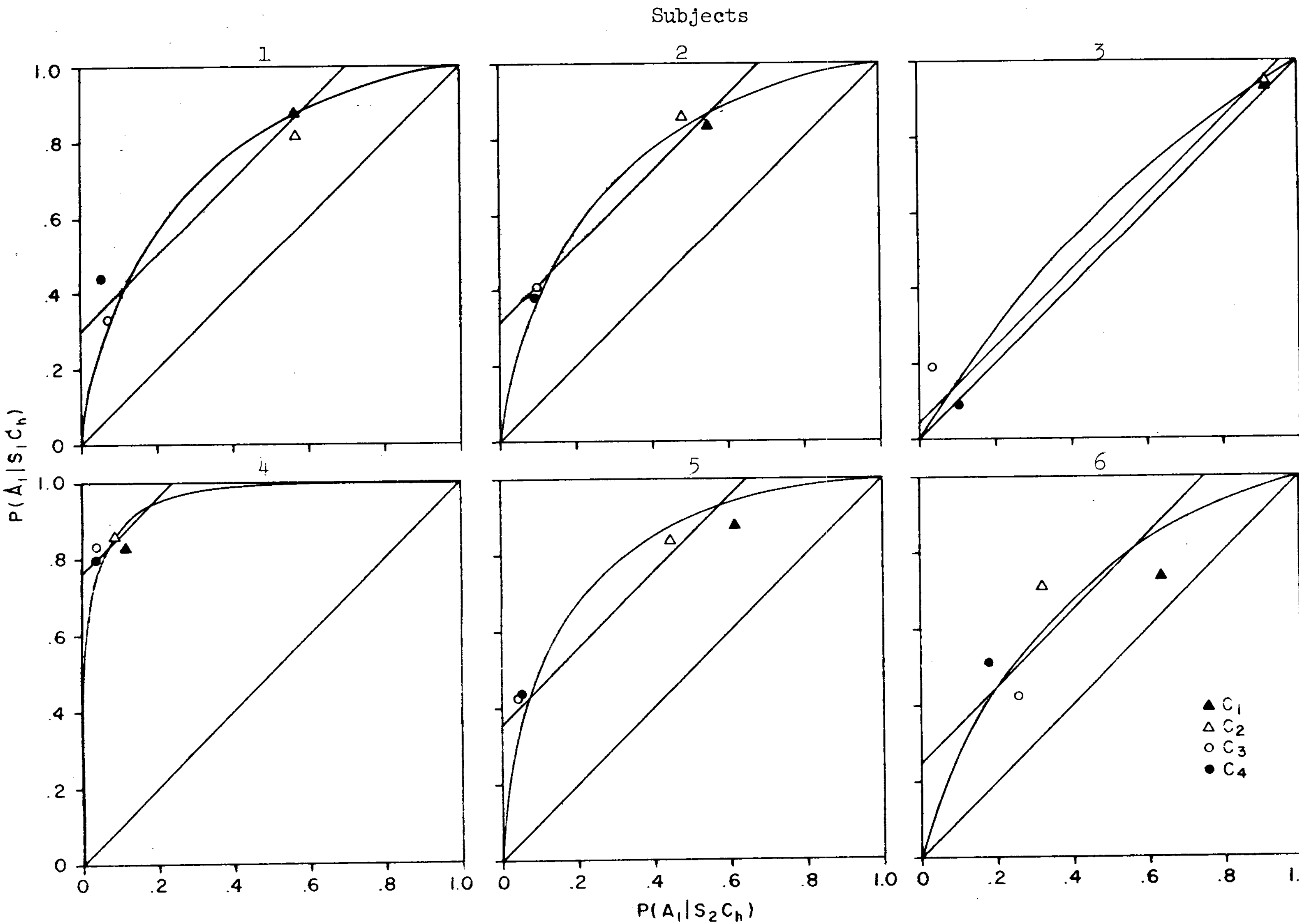

Figure 8a. Recognition-Confusion and Signal Detectability One-Parameter ROC Curves for Group 2.

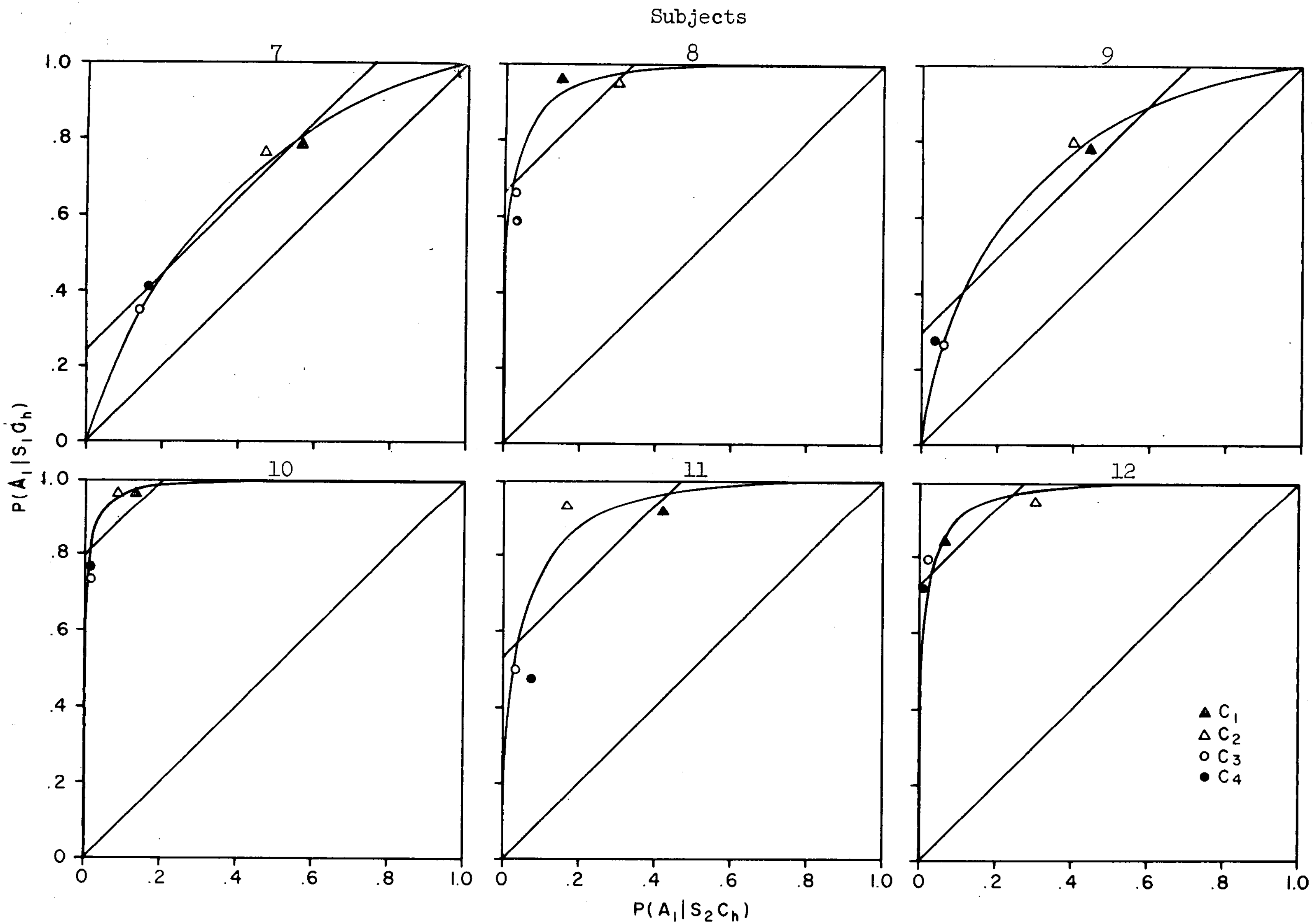

Figure 8b. Recognition-Confusion and Signal Detectability One-Parameter ROC Curves for Group 2.

Table 5

Sum of Squared Deviations for the

Straight Line and Curvilinear Fits.

Subject	Group 1 Curvilinear	Group 1 Straight Line	Group 2 Curvilinear	Group 2 Straight Line
1	.008	.010	.008	.006
2	.001	.006	.002	.003
3	.005	.006	.008	.009
4	.002	.000	.003	.006
5	.012	.010	.007	.006
6	.010	.002	.024	.028
7	.001	.001	.002	.002
8	.002	.001	.003	.018
9	.003	.001	.002	.012
10	.002	.015	.001	.008
11	.006	.009	.010	.039
12	.002	.003	.002	.006
Averages	.0045	.0057	.0060	.0119

better for a majority of subjects in Group 2. In fact, a paired t-test showed a significant difference in the two types of fit for Group 2 but not for Group 1 $(P = .05)$. This is somewhat surprising in view of the finding that the sensitivity index was significantly larger for the middle cues in Group 1 but not in Group 2: larger σ values for the middle cues would be expected to enhance the appearance of curvilinearity in the ROC space. A straight line of slope greater than one would probably do much better for several of the subjects (subjects 8, 9, 10, 11 especially) in Group 1 than does the straight line of slope one.

Latencies

Figure 9 presents the mean latencies plotted against γ for the two groups. There are several aspects of these data which bear comment.

The latencies seem to differ according to which stimulus type was displayed, and there is a crossover of the S_1 and S_2 latencies, with the S_1 latencies being longer than the S_2 latencies when $\gamma < \frac{1}{2}$ and shorter than the S_2 latencies when $\gamma > \frac{1}{2}$. The latencies conditional on the response made by the subject also show a crossover: the A_1 latencies are longer than the A_2 latencies when $\gamma < \frac{1}{2}$ and shorter than the A_2 latencies when $\gamma > \frac{1}{2}$.

Conditionalizing on the joint event of an A_iS_j reveals a crossover effect of the type noted for the A_i and S_j latencies, both for correct and incorrect responses. The fact that the A_1S_2 and A_2S_1 latencies follow the same general form as the A_1 and A_2 latencies respectively, suggests the possibility that the differential stimulus effect is low on incorrect trials.

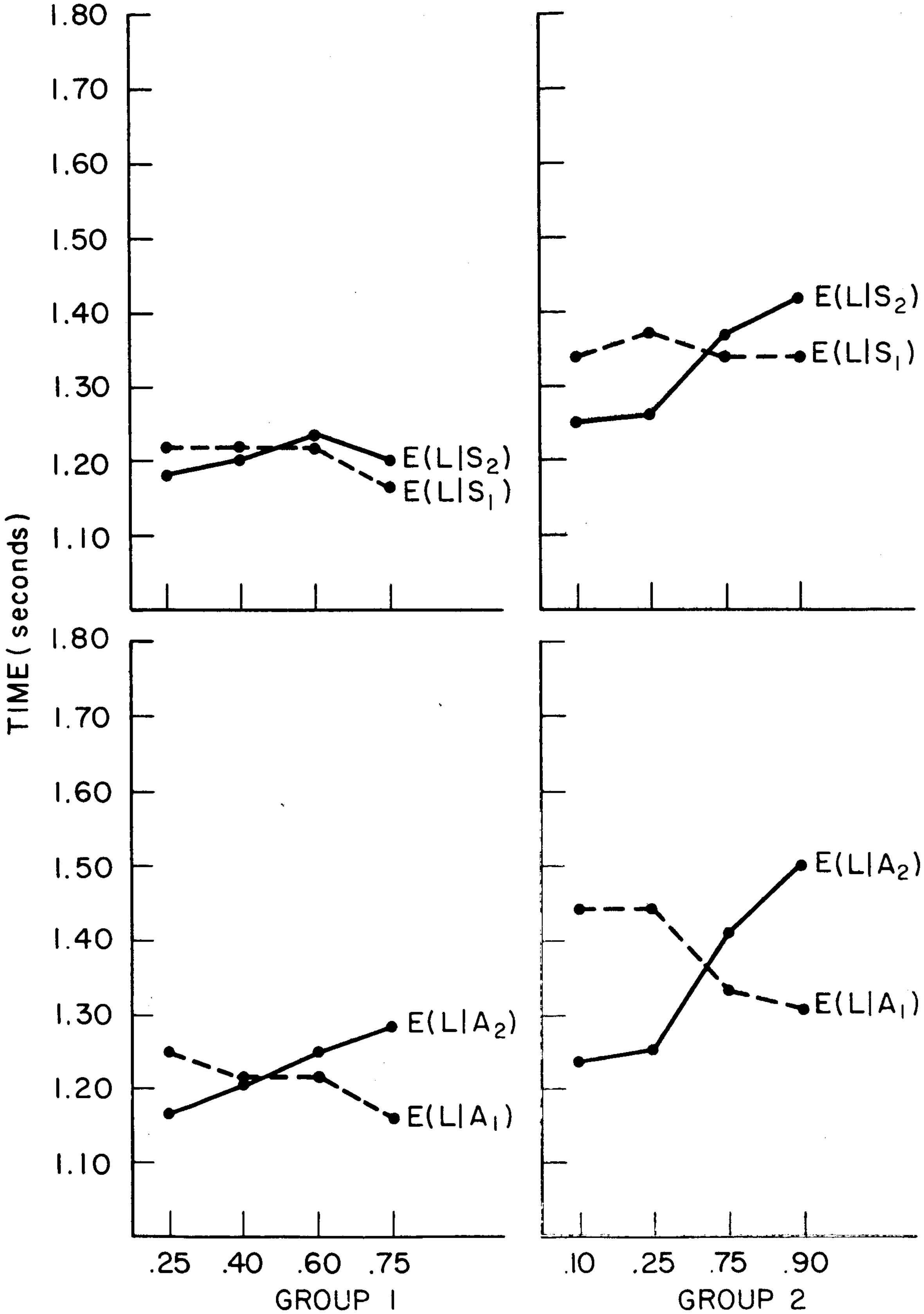

Figure 9a. Mean Latencies as Functions of γ.

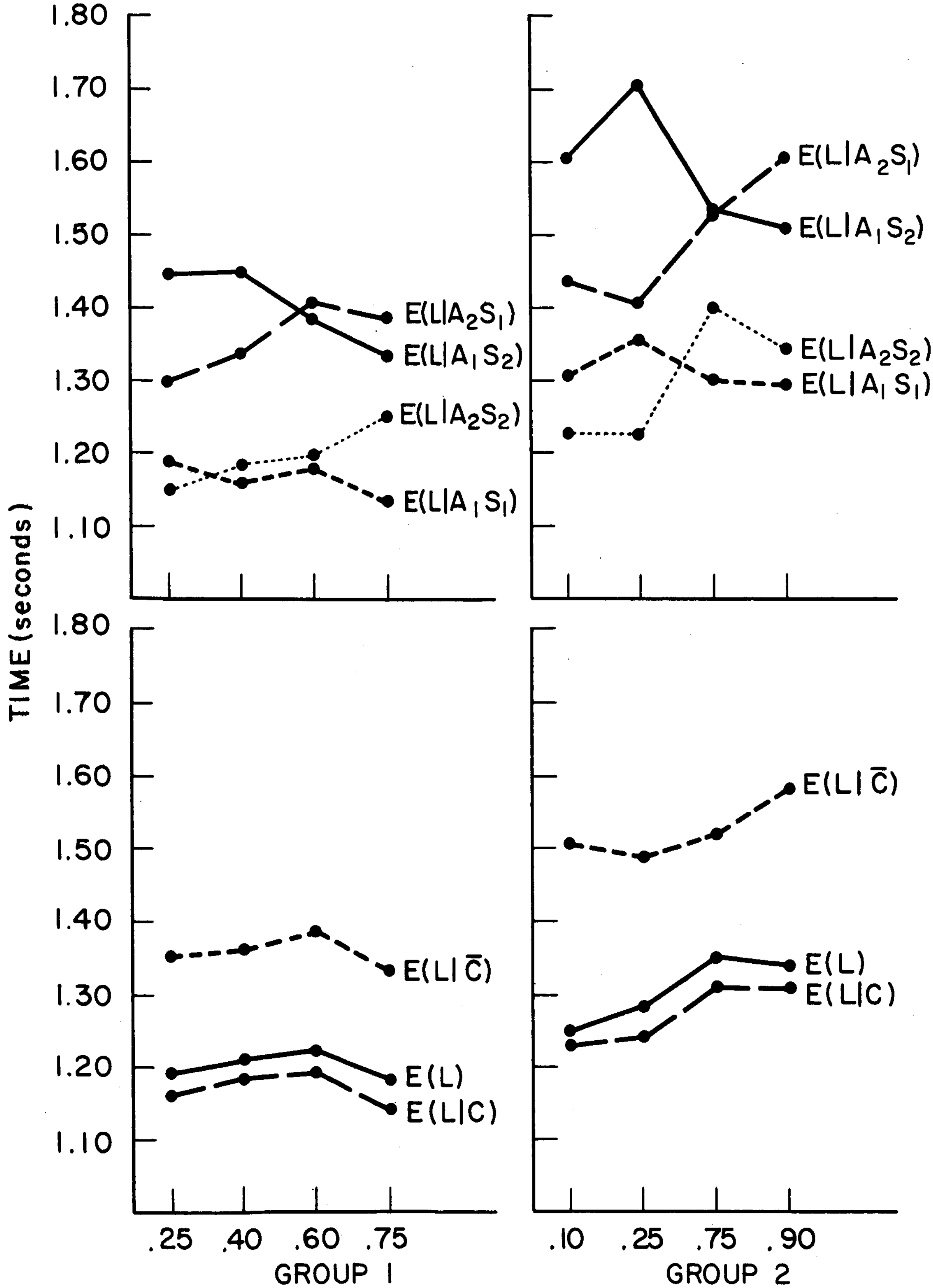

Figure 9b. Mean Latencies as Functions of γ.

Correct responses are associated with shorter response times on the average in these data for both groups, but the overall average latencies appear to be longer when $\gamma > \frac{1}{2}$ for Group 2, with no appreciable difference evident for Group 1.

Finally, the members of Group 2 seem to have responded more slowly on the whole than did those of Group 1, but the difference was non-significant according to an independent t-test.

Confidence Ratings

Figure 10 presents $P(A_1|S_1)$ and $P(A_1|S_2)$ as functions of the confidence rating, where CR_k refers to confidence rating number k. It will be recalled that there were four confidence ratings with CR_1 representing the most confident response possible, ranging down to CR_4 as the confidence rating the subject was instructed to give when he felt he was guessing at random. The major effect to be noted is a general regression of $P(A_1|S_1)$ and $P(A_1|S_2)$ toward $\frac{1}{2}$ as the confidence rating went from 1 to 3; at CR_4 there is an increase or decrease in the proportion of A_1 responses made independent of whether an S_1 or an S_2 was presented. If $\gamma > \frac{1}{2}$, the proportion of A_1 responses increased given CR_4; and if $\gamma < \frac{1}{2}$, the proportion of A_1 responses decreased given CR_4. Thus we might infer that the subjects were able to grade their performance in an effective manner employing CR_1, CR_2, and CR_3 to rank their accuracy in decisions that were made on a sensory basis. Performance on CR_4, on the other hand, appears to reflect the subjects' response biases. Although behavior in Groups 1 and 2 was highly similar, Group 2 seems to have used CR_2 in a way slightly different than did Group 1. A slight increase in the A_2 bias seems to occur in Group 2 given CR_2.

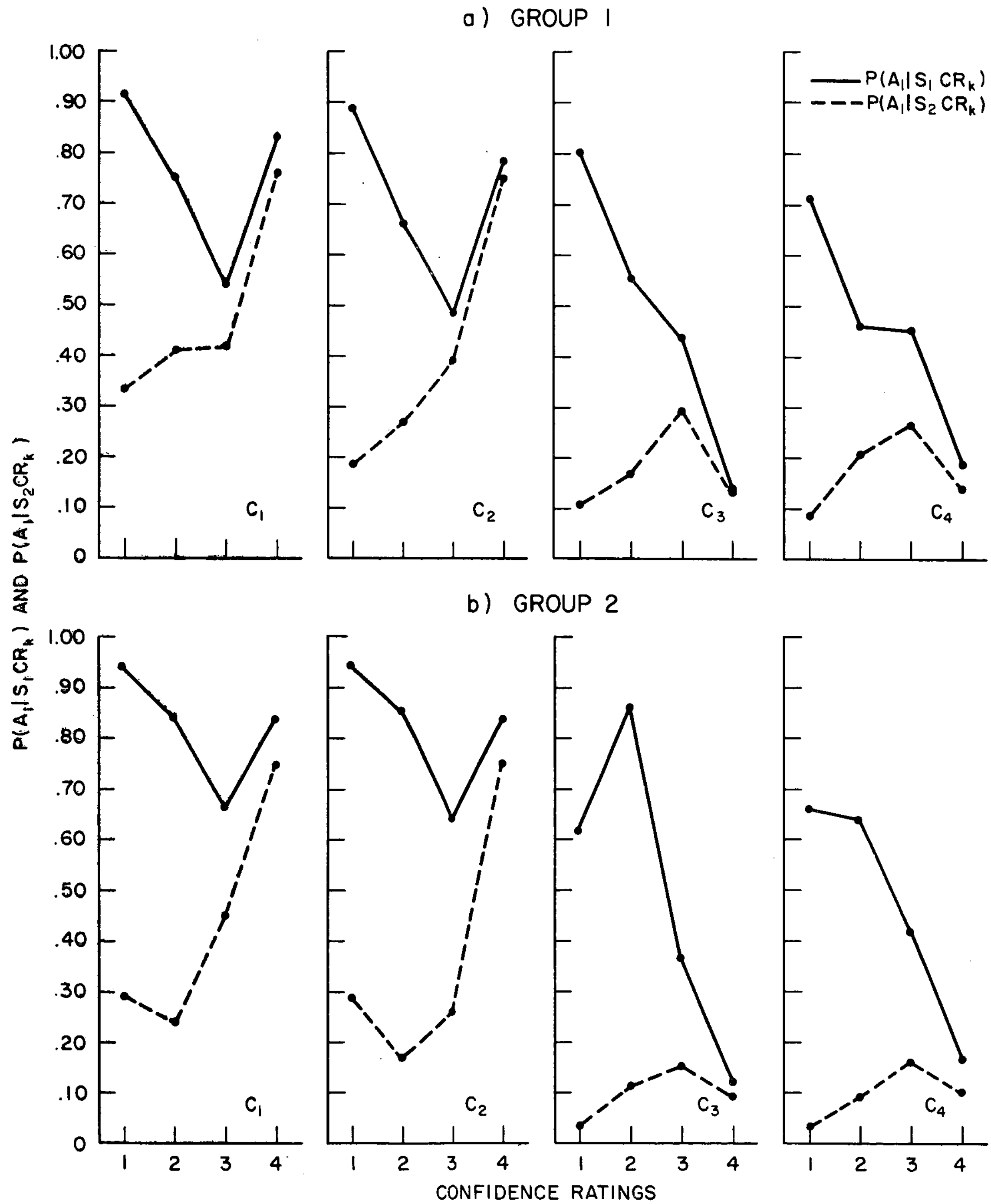

Figure 10. $P(A_1|S_1)$ and $P(A_1|S_2)$ as Functions of CR_k.

In this section, several cases of the general recognition-confusion model described earlier will be applied to the ROC data with the aim of specifying those models that correspond with the present experiment. First, Cases 1 and 2 as developed earlier (page 17) will be tested against the data and compared with one another as to goodness of fit. Then, one of these cases will be employed to investigate whether the result $P(A_1) \neq \frac{1}{2}$ seems to follow from an asymmetry in confusability or an asymmetry in the guessing bias. Finally to be considered are two cases that assume, in contrast to Cases 1 and 2, that either the signals are confused with one another, or with the noise symbols, rather than assuming that the noise symbols are confused with the signals.

Case 1 and Case 2

Since Cases 1 and 2 were derived in detail earlier, it will suffice here to present their associated activation matrices. The activation matrix for Case 1 is

$$N_1 = \begin{array}{c} \\ Z_0 \\ Z_1 \\ Z_2 \end{array} \begin{array}{c} \begin{array}{ccc} s_0 & s_1 & s_2 \end{array} \\ \begin{bmatrix} u & (1-u)\frac{1}{2} & (1-u)\frac{1}{2} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array},$$

and that for Case 2 is

$$
\begin{array}{ccc}
 & & \begin{array}{ccc} s_0 & \quad s_1 \quad & s_2 \end{array} \\
N_1 & = & \begin{array}{c} Z_0 \\ Z_1 \\ Z_2 \end{array} \left[\begin{array}{ccc} v^i & (1-v^i)\frac{1}{2} & (1-v^i)\frac{1}{2} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right] .
\end{array}
$$

Each of the two models has six free parameters: the activation parameter, u or v; the sample size d; the four bias parameters, g_1, g_2, g_3, and g_4. The method of estimation for each subject consisted of consecutively setting d equal to 1, 2, 2, ... , 15, 16; for each of these d values the sensitivity index, a function of u (Case 1) or v (Case 2) and d, was set equal to the intercept of the straight line obtained by orthogonal regression and the resultant "equation" solved for u or v. For some values of d, the only solution to the equation was a u or v greater than one; when this occurred, the parameter u or v was set equal to 1. Next, the guessing bias for each of the four points in the ROC space was obtained from the expression for $P(A_1|C_h)$ (involving u or v, g_h and d and the observed value for this quantity. The six estimated parameters were then used to predict $P(A_1|S_2)$ and $P(A_1|S_1)$ for the four cues after which the sum of the squared deviations of the observed points from the predicted points was calculated: $\sum_{h=1}^{4} \{[P^{(t)}(A_1|S_2C_h) - P^{(o)}(A_1|S_2C_h)]^2 + [P^{(t)}(A_1|S_1C_h) - P^{(o)}(A_1|S_1C_h)]^2\}$ where t refers to the theoretical or predicted value and o to the observed value. Thus, for each value of d from 1 to 16, values of the other five parameters were obtained and used to provide a fit to the four points in the ROC space. After this was accomplished for each value of d, that set of parameter values that yielded a minimum sum of squared deviations of predicted from observed points was selected.

Tables 6 and 7 present means and standard errors for the estimated parameters, for the predicted and observed coordinates in the ROC space and for the sum of the squared deviations of the predicted from the observed points. The fits for Groups 1 and 2 are presented separately.

The most striking feature of these data is that Cases 1 and 2 essentially reduce to the same model. That is, when $u = v = 1$, N_1 becomes the identity matrix and the two cases are equivalent. Only three subjects out of twelve in Group 1 and two out of twelve in Group 2 had $u \neq 1$. These u values were .94, .96, and .97 for the Group 1 subjects and .98 for both the subjects in Group 2. Estimated v values were 1 for all twelve subjects in each group. Under the assumptions of the model, this result implies that there was a negligible amount of confusion of the noise symbols with the signals.

A second interesting result is that the estimates of the bias parameters reflect much more strongly than did $P(A_1)$ (averaged over subjects in each group) the apparent tendency to respond A_2 more often than A_1. Since for this analysis $P(A_1|S_2)$ and $P(A_1|S_1)$ reduce to

$$P(A_1|S_2) = (1-\sigma)g \ ,$$

$$P(A_1|S_1) = \sigma + (1-\sigma)g \ ,$$

where $\sigma = d/D$, the difference in $P(A_1)$, corresponding to a difference in the guessing bias of $g_1 - g_2$, is $P_1(A_1) - P_2(A_1) = (1-\sigma)\cdot(g_1-g_2)$. Hence, an attenuated difference in $P(A_1)$ is expected.

Tables 6a and 7a indicate that the C_1 and C_4 points were fit somewhat better than the C_2 and C_3 points. Finally, it is interesting that the standard errors of the observed points are closely approximated by the standard errors of the predicted points (Tables 6b, 7b).

<u>Case 1a and Case 1b</u>

The result $(g_1 + g_2 + g_3 + g_4)/4 < \frac{1}{2}$ may follow from an asymmetry in response bias or it may be due to the noise symbols being more easily confusable with $\ominus$ than with Φ. Since Cases 1 and 2 fit equally well, the simpler Case 1 will be used here to investigate whether either of the two hypotheses (response bias vs. confusability) is favored over the other.

To evaluate the proposition that there was an asymmetry in confusion, it was assumed that the subjects' probability matched $(g_h = \gamma_h)$ but that $q \neq \frac{1}{2}$, i.e., that the likelihood of confusing a noise symbol with signal symbol Z_1, Φ, was not the same as the likelihood of confusing a noise symbol with signal symbol $Z_2, \ominus$. Three parameters then remained to be estimated: d, u, and q. This model will be denoted Case 1a. Its activation matrix is

$$N_i = \begin{array}{c} \\ Z_0 \\ Z_1 \\ Z_2 \end{array} \begin{array}{c} \begin{array}{ccc} s_0 & s_1 & s_2 \end{array} \\ \begin{bmatrix} u & (1-u)q & (1-u)(1-q) \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array}.$$

A model that will be referred to as Case 1b was used to obtain a fit under the hypothesis that an asymmetry existed in the efficacy of E_1 and E_2. Employing the simple linear model on the guessing bias:

Table 6a.

Means Associated with Case 1 and Case 2

Fits to ROC Data for Group 1.

Parameters

	d	u	g_1	g_2	g_3	g_4
Case 1	6.667	.988	.702	.516	.228	.191
Case 2	6.417	1.000	.679	.559	.264	.196

ROC Points and Goodness-of-Fit Measure

	C_1		C_2		C_3		C_4		
	$P(A_1\|S_2)$	$P(A_1\|S_1)$	$P(A_1\|S_2)$	$P(A_1\|S_1)$	$P(A_1\|S_2)$	$P(A_1\|S_1)$	$P(A_1\|S_2)$	$P(A_1\|S_1)$	$\Sigma(\text{DEV})^2$
Case 1	.437	.837	.340	.755	.163	.576	.133	.533	.008
Case 2	.436	.837	.349	.750	.169	.570	.132	.533	.008
Observed	.431	.839	.318	.770	.159	.583	.135	.525	

Table 6b.

Standard Errors of the Means Associated

with Case 1 and Case 2 Fits to ROC Data for Group 1.

Parameters

	d	u	g_1	g_2	g_3	g_4
Case 1	1.163	.006	.065	.067	.047	.040
Case 2	1.163	.000	.059	.051	.043	.040

ROC Points and Goodness-of-Fit Measure

	C_1		C_2		C_3		C_4		
	$P(A_1\|S_2)$	$P(A_1\|S_1)$	$P(A_1\|S_2)$	$P(A_1\|S_1)$	$P(A_1\|S_2)$	$P(A_1\|S_1)$	$P(A_1\|S_2)$	$P(A_1\|S_1)$	$\Sigma(\text{DEV})^2$
Case 1	.081	.033	.065	.039	.046	.049	.040	.053	.002
Case 2	.081	.034	.064	.041	.046	.052	.040	.053	.002
Observed	.079	.036	.053	.044	.038	.057	.042	.055	

Table 7a.

Means Associated with Case 1 and Case 2

Fits to ROC Data for Group 2.

Parameters

	d	u	g_1	g_2	g_3	g_4
Case 1	7.083	.997	.731	.757	.112	.103
Case 2	6.833	1.000	.725	.755	.115	.111

ROC Points and Goodness-of-Fit Measure

	C_1		C_2		C_3		C_4		
	$P(A_1 \mid S_2)$	$P(A_1 \mid S_1)$	$P(A_1 \mid S_2)$	$P(A_1 \mid S_1)$	$P(A_1 \mid S_2)$	$P(A_1 \mid S_1)$	$P(A_1 \mid S_2)$	$P(A_1 \mid S_1)$	$\Sigma(DEV)^2$
Case 1	.429	.869	.425	.866	.070	.511	.070	.511	.020
Case 2	.439	.866	.438	.864	.072	.498	.073	.500	.021
Observed	.434	.865	.383	.869	.071	.488	.071	.502	

Table 7b.

Standard Errors of the Means Associated

with Case 1 and Case 2 Fits to ROC Data for Group 2.

Parameters

	d	u	g_1	g_2	g_3	g_4
Case 1	1.121	.002	.054	.035	.023	.020
Case 2	1.135	.000	.053	.035	.035	.020

ROC Points and Goodness-of-Fit Measure

	C_1		C_2		C_3		C_4		
	$P(A_1\|S_2)$	$P(A_1\|S_1)$	$P(A_1\|S_2)$	$P(A_1\|S_1)$	$P(A_1\|S_2$	$P(A_1\|S_1)$	$P(A_1\|S_2)$	$P(A_1\|S_1)$	$\Sigma(DEV)^2$
Case 1	.067	.020	.060	.023	.019	.063	.017	.060	.005
Case 2	.067	.020	.061	.023	.019	.063	.017	.060	.005
Observed	.065	.021	.047	.024	.020	.056	.016	.050	

Learning function	Stimulus type	Feedback	Probability
$g_{n,h} = (1-\theta)g_{n-1,h} + \theta$	$S_{1,n-1,h}$	$E_{1,n-1,h}$	γ_h
$g_{n,h} = (1-\theta')g_{n-1,h}$	$S_{2,n-1,h}$	$E_{2,n-1,h}$	$1-\gamma_h$

where n refers to trial number. It can then be shown that

$$\lim_{n \to \infty} g_{n,h} = \frac{\gamma_h}{\gamma_h+(1-\gamma_h)\theta'/\theta} = \frac{\gamma_h}{\gamma_h+(1-\gamma_h)\varphi} .$$

Hence, rather than estimating the g_h values separately, we can reduce the number of parameters to three (d, u, φ) and at the same time obtain an index of the relative effectiveness of E_2 and E_1 (φ). In general we would expect to find $\varphi > 1$ in the present data since this inequality would imply a greater bias for the A_2 response. The activation matrix for this model is identical to that of Case 1.

The method of estimation was similar to that used for Cases 1 and 2; the only difference was that φ and q were estimated for each subject from expressions containing the overall average (over cues) of $P(A_1)$. For Case 1a,

$$q = \frac{1}{4}\left\{\frac{\sum_{h=1}^{4} P(A_1|C_h) - 2[(1-\frac{d}{D})u^d + \frac{1}{D}\frac{1-u^d}{1-u}]}{1 - (1-\frac{d}{D})u^d - \frac{1}{D}\frac{1-u^d}{1-u}}\right\},$$

and for Case 1b,

$$\varphi = \frac{1}{4}\sum_{h=1}^{4}\frac{\gamma_h}{1-\gamma_h}\left\{\frac{u^d(1-\frac{d}{D})}{P(A_1|C_h) - \frac{1}{2}[1-(1-\frac{d}{D})u^d - \frac{1}{D}\frac{1-u^d}{1-u}] - \gamma_h\frac{1-u^d}{1-u}} - 1\right\}.$$

The results of these fits are presented in Tables 8 and 9 in a manner comparable to that used for Cases 1 and 2.

Table 8 shows a superiority in terms of $\sum(\text{DEV})^2$ of Case 1a over Case 1b for Group 1. However, this is offset by the fact that one subject in Group 1 (subject 5) had to be excluded from the data in Table 8, since there was no set of parameters satisfying the constraints of probability measurement that could be estimated for that subject. Also, after subject 5 was deleted, there were still six subjects who were fit better by Case 1b, as opposed to five that were fit better by Case 1a.

Group 2 subjects (Table 9) are fit better by both models than are the Group 1 subjects, and further, Case 1b fits Group 2 better than Case 1a in terms of $\sum(\mathrm{DEV})^2$ and in terms of the number of subjects (excluding subject 3, who could not be fit with Case 1a) fit better by Case 1b (seven out of eleven). The observed averages and standard errors in Tables 8 and 9 excluded subjects 7 and 3 in Groups 1 and 2, respectively.

The reader should note that, as was expected, the fits were substantially better when six parameters were estimated from the data (Cases 1 and 2). Also, the average values of q and φ clearly reflect the asymmetry in $P(A_1|C_h)$.

Case 3 and Case 4

Cases 1 and 2 were based on the proposition that confusion occurred when a Z_0 was processed but not when a signal was processed. In this section two cases that include an alternative assumption will be investigated; namely, confusion may result from the processing of a signal symbol but not from the processing of a noise symbol. Case 3 posits that processing a signal symbol can lead to an s_0 activation but not an activation of the hypothetical sensory state of the alternative signal. Thus,

$$N_i = \begin{array}{c} \\ Z_0 \\ Z_1 \\ Z_2 \end{array} \begin{array}{c} \begin{array}{ccc} s_0 & s_1 & s_2 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 \\ 1-a & a & 0 \\ 1-a' & 0 & a' \end{bmatrix} \end{array} .$$

Table 8a.

Means Associated with Case 1a and Case 1b

Fits to ROC Data for Group 1.

Parameters

	d	u	q	φ	g_1	g_2	g_3	g_4
Case 1a	8.545	.922	.266		.750	.600	.400	.250
Case 1b	6.818	.989		.273	.567	.413	.254	.153

ROC Points and Goodness-of-Fit Measure

	C_1		C_2		C_3		C_4		
	$P(A_1\|S_2)$	$P(A_1\|S_1)$	$P(A_1\|S_2)$	$P(A_1\|S_1)$	$P(A_1\|S_2)$	$P(A_1\|S_1)$	$P(A_1\|S_2)$	$P(A_1\|S_1)$	$\Sigma(DEV)^2$
Case 1a	.308	.761	.262	.716	.202	.656	.156	.610	.080
Case 1b	.350	.758	.268	.675	.179	.587	.121	.529	.096
Observed	.388	.832	.280	.761	.129	.598	.123	.545	

Table 8b.

Standard Errors of the Means Associated

with Case 1a and Case 1b Fits to ROC Data for Group 1.

Parameters

	d	u	q	φ	g_1	g_2	g_3	g_4
Case 1a	1.269	.030	.063		.000	.000	.000	.000
Case 1b	1.304	.006		.522	.046	.048	.041	.031

ROC Points and Goodness-of-Fit Measure

	C_1		C_2		C_3		C_4		
	$P(A_1\|S_2)$	$P(A_1\|S_1)$	$P(A_1\|S_2)$	$P(A_1\|S_1)$	$P(A_1\|S_2)$	$P(A_1\|S_1)$	$P(A_1\|S_2)$	$P(A_1\|S_1)$	$\Sigma(DEV)^2$
Case 1a	.057	.029	.050	.033	.043	.041	.039	.048	.039
Case 1b	.061	.037	.053	.047	.042	.056	.031	.063	.058
Observed	.079	.036	.053	.044	.038	.057	.042	.055	

Table 9a.

Means Associated with Case 1a and Case 1b

Fits to ROC Data for Group 2.

Parameters

	d	u	q	φ	g_1	g_2	g_{3n}	g_4
Case 1a	8.909	.948	.305		.750	.900	.100	.250
Case 1b	7.545	.986		3.70	.530	.752	.046	.124

ROC Points and Goodness-of-Fit Measure

	C_1		C_2		C_3		C_4		
	$P(A_1\|S_2)$	$P(A_1\|S_1)$	$P(A_1\|S_2)$	$P(A_1\|S_1)$	$P(A_1\|S_2)$	$P(A_1\|S_1)$	$P(A_1\|S_2)$	$P(A_1\|S_1)$	$\Sigma(DEV)^2$
Case 1a	.303	.779	.348	.825	.106	.582	.151	.628	.066
Case 1b	.312	.767	.419	.874	.052	.507	.095	.550	.053
Observed	.386	.854	.331	.861	.074	.515	.068	.536	

Table 9b.

Standard Errors of the Means Associated

with Case 1a and Case 1b Fits to ROC Data for Group 2.

Parameters

	d	u	q	φ	g_1	g_2	g_3	g_4
Case 1a	1.195	.012	.039		.000	.000	.000	.000
Case 1b	1.101	.010		1.22	.045	.042	.066	.015

ROC Points and Goodness-of-Fit Measure

	C_1		C_2		C_3		C_4		
	$P(A_1\|S_2)$	$P(A_1\|S_1)$	$P(A_1\|S_2)$	$P(A_1\|S_1)$	$P(A_1\|S_2)$	$P(A_1\|S_1)$	$P(A_1\|S_2)$	$P(A_1\|S_1)$	$\Sigma(DEV)^2$
Case 1a	.044	.024	.052	.020	.017	.053	.021	.045	.011
Case 1b	.043	.029	.056	.022	.015	.061	.016	.054	.011
Observed	.065	.021	.047	.024	.020	.056	.016	.050	

Case 4, on the other hand, supposes that the two signals may be confused with one another but never with a Z_0 symbol. In this case,

$$N_i = \begin{array}{c} \\ Z_0 \\ Z_1 \\ Z_2 \end{array} \begin{array}{c} \begin{array}{ccc} s_0 & s_1 & s_2 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 \\ 0 & a & 1-a \\ 0 & 1-a' & a' \end{bmatrix} \end{array} .$$

Estimation for Case 3 was accomplished by stepping d from 1 to 16 and for each d setting $a = (I/d)\cdot 16$ unless $(I/d)\cdot 16 > 1$, in which case $a = 1$; I was the intercept of the straight line (of slope 1) obtained by the method of least squares. Then a' was determined from

$$a' = \frac{4}{d}\left\{4 - (1 - \frac{ad}{16})\sum_{h=1}^{4} \frac{16\ P(A_1|S_2C_h)}{16\ P(A_1|S_1C_h) - ad}\right\}$$

and φ from

$$\varphi = \frac{1}{4}\sum_{h=1}^{4}\left\{\frac{(\frac{\gamma_h}{1-\gamma_h})[1-(1-\gamma_h)\frac{a'd}{16} - \gamma_h \frac{ad}{16}]}{P(A_1|C_h) - \gamma_h \frac{ad}{16}} - \frac{\gamma_h}{1-\gamma_h}\right\} .$$

For the ROC analysis a and d are tied together in the expression $\frac{ad}{16}$ and a' and d are tied together in the expression $\frac{a'd}{16}$. We can let $ad/16 = \sigma_1$ and $a'd/16 = \sigma_2$ and argue that in essence, only two parameters are being estimated here plus one more for the estimate of φ. As for Cases 1 and 2, those parameter values that yielded a minimum sum of deviations of observed points from theoretical points were selected for each subject.

For Case 4, d is again run in steps of 1 from 1 to 16; for each of these values

$$a = \left[\frac{\sum_{h=1}^{4} P(A_1|S_1C_h)}{4} - (1 - \frac{d}{16})\frac{1}{2}\right]\frac{16}{d}, \quad a' = \left[(1 - \frac{d}{16})\frac{1}{2} - \frac{\sum_{h=1}^{4} P(A_1|S_2C_h)}{4}\right]\frac{16}{d} + 1 .$$

Here the parameters a, d and a', d are distinct in the expressions for $P(A_1|S_2)$ and $P(A_1|S_1)$ and hence are associated with three degrees of freedom. We therefore set $g_h = \gamma_h$ for this case. Again, those parameter values estimated in this manner are selected that minimize $\Sigma(DEV)^2$.

Tables 10 and 11 give the means and standard errors of the parameter estimates, and the predicted and observed points in the ROC space. As estimated for both groups, φ again reflects the bias to the A_2 response, although not so dramatically as in Case 1b; this probably results from the capability of σ_1 and σ_2 to reflect the A_2 bias. The parameters a and a' in Case 4 also predict the $P(A_1)$ asymmetry, although through intersignal confusion instead of signal-noise confusion.

Table 12 presents the goodness-of-fit measure for individuals in the four conditions. Note that as predicted earlier subjects 8, 9, 10 and 11 of Group 2 are fit much better by Case 3 than by any of the other cases. This is due to a slope greater than one evident in their ROC data. Table 13 indicates that in terms of the number of subjects fit best, Case 3 provides the best description for Group 1, but Case 4 is best for Group 2. Overall, there is a tie between Case 3 and Case 4. The second part of Table 13 shows the average of $\Sigma(DEV)^2$ over subjects (excluding subject 5 in Group 1 and subject 3 in Group 2); of the three parameter models, Case 4 was supercendent for both groups. Thus, of the three-parameter models, Case 4 provides the best description of the data. Finally, it should be remarked that in addition to providing a reasonable fit to the data in terms of $\Sigma(DEV)^2$ for each subject, the models appear to do quite well in fitting the group means. In particular, the approximations of the means of the predicted values to the means of the observed values are quite striking for Cases 1 and 2.

Table 10a.

Means Associated with Case 3 and Case 4

Fits to ROC Data for Group 1.

	Parameters									
	σ_1	σ_2	φ	d	a	a'	g_1	g_2	g_3	g_4
Case 3	.383	.535	1.59				.674	.520	.338	.211
Case 4				10.545	.768	.894	.750	.600	.400	.250

ROC Points and Goodness-of-Fit Measure

	C_1		C_2		C_3		C_4		
	$P(A_1\|S_2)$	$P(A_1\|S_1)$	$P(A_1\|S_2)$	$P(A_1\|S_1)$	$P(A_1\|S_2)$	$P(A_1\|S_1)$	$P(A_1\|S_2)$	$P(A_1\|S_1)$	$\Sigma(DEV)^2$
Case 3	.312	.804	.242	.710	.167	.599	.101	.520	.089
Case 4	.315	.769	.264	.718	.196	.650	.145	.599	.075
Observed	.388	.832	.280	.761	.129	.598	.123	.545	

Table 10b.

Standard Errors of the Means Associated

with Case 3 and Case 4 Fits to ROC Data for Group 1.

Parameters

	σ_1	σ_2	φ	d	a	a'	g_1	g_2	g_3	g_4
Case 3	.073	077	.258				.035	.040	.038	.030
Case 4				.930	.041	.062	.000	.000	.000	.000

ROC Points and Goodness-of-Fit Measure

	C_1		C_2		C_3		C_4		
	$P(A_1\|S_2)$	$P(A_1\|S_1)$	$P(A_1\|S_2)$	$P(A_1\|S_1)$	$P(A_1\|S_2)$	$P(A_1\|S_1)$	$P(A_1\|S_2)$	$P(A_1\|S_1)$	$\Sigma(DEV)^2$
Case 3	.060	.030	.053	.040	.041	.050	.030	.056	.051
Case 4	.059	.030	.051	.033	.042	.040	.037	.047	.038
Observed	.079	.036	.053	.044	.038	.057	.042	.055	

Table 11a.

Means Associated with Case 3 and Case 4

Fits to ROC Data for Group 2.

Parameters

	σ_1	σ_2	φ	d	a	a'	g_1	g_2	g_3	g_4
Case 3	.419	.540	1.85				.645	.837	.072	.184
Case 4				10.182	.785	.945	.750	.900	.100	.250

ROC Points and Goodness-of-Fit Measure

	C_1		C_2		C_3		C_4		
	$P(A_1\|S_2)$	$P(A_1\|S_1)$	$P(A_1\|S_2)$	$P(A_1\|S_1)$	$P(A_1\|S_2)$	$P(A_1\|S_1)$	$P(A_1\|S_2)$	$P(A_1\|S_1)$	$\Sigma(DEV)^2$
Case 3	.294	.798	.385	.910	.030	.460	.079	.526	.061
Case 4	.306	.782	.360	.837	.069	.546	.124	.600	.047
Observed	.386	.854	.331	.861	.074	.515	.068	.536	

Table 11b.

Standard Errors of the Means Associated

with Case 3 and Case 4 Fits to ROC Data for Group 2.

Parameters

	σ_1	σ_2	φ	d	a	a'	g_1	g_2	g_3	g_4
Case 3	.062	.072	.349				.035	.023	.010	.022
Case 4				.943	.032	.022	.000	.000	.000	.000

ROC Points and Goodness-of-Fit Measure

	C_1		C_2		C_3		C_4		
	$P(A_1\|S_2)$	$P(A_1\|S_1)$	$P(A_1\|S_2)$	$P(A_1\|S_1)$	$P(A_1\|S_2)$	$P(A_1\|S_1)$	$P(A_1\|S_2)$	$P(A_1\|S_1)$	$\Sigma(DEV)^2$
Case 3	.046	.025	.060	.012	.005	.059	.012	.053	.019
Case 4	.046	.022	.054	.018	.014	.055	.020	.047	.008
Observed	.065	.021	.047	.024	.020	.056	.016	.050	

Table 12.

$\Sigma(\mathrm{DEV})^2$ for Group 1 and Group 2 Subjects.

	Group 1				Group 2			
Subject	Case 1a	Case 1b	Case 3	Case 4	Case 1a	Case 1b	Case 3	Case 4
1	.065	.049	.043	.039	.035	.041	.049	.034
2	.066	.082	.051	.065	.078	.035	.039	.034
3	.008	.009	.019	.007	---	.211	---	.176
4	.003	.006	.040	.003	.003	.006	.014	.003
5	---	.080	.065	.139	.063	.086	.097	.061
6	.014	.008	.008	.013	.082	.136	.247	.081
7	.004	.001	.001	.001	.060	.050	.069	.024
8	.014	.006	.004	.012	.061	.023	.014	.038
9	.035	.023	.023	.028	.123	.062	.037	.075
10	.436	.689	.601	.433	.045	.020	.004	.034
11	.232	.172	.182	.214	.137	.096	.056	.103
12	.005	.008	.009	.005	.041	.024	.040	.034

Table 13.

Number of Subjects Fit Best By Each

of the Three Parameter Models

(Subjects with one or more ties for closest fit were omitted.)

	Group 1	Group 2	Total
Case 1a	2	0	2
Case 1b	1	1	2
Case 3	3	4	7
Case 4	1	6	7

Average $\Sigma(\mathrm{DEV})^2$

	6 Parameters		3 Parameters			
	Case 1	Case 2	Case 1a	Case 1b	Case 3	Case 4
Group 1	.008	.008	.080	.096	.089	.075
Group 2	.020	.021	.066	.066	.061	.047

DISCUSSION

The result of the comparison of the straight line and the curvilinear fits to the ROC results was that they did equally well for Group 1, but the straight line provided a significantly inferior fit to that of the curved line for Group 2. It may be that there is a strong element of curvilinearity in the Group 2 data that is unrelated to a higher sensitivity on the less biased cues (in terms of variable sensitivity notions). However, there is an aspect of the data that argues against this hypothesis. Although sensitivity for the low-correlated cues did not differ significantly from sensitivity for the high-correlated cues for Group 2, it can be seen from Table 5 and Table 14 that those subjects who contributed most heavily to the poorer performance by the straight-line fit (primarily subjects 8, 9, 10, and 11) had larger sensitivity indices associated with one or more of their lower biased cues than for their higher biased cues, <u>and</u> their higher biased cue-points tended to be closer to the axes than was the case for other subjects. Thus, the source of the difference in fit for the straight line and curved line was a difference in sensitivity; furthermore, the resulting set of points could be fit better by a curved line than by a straight line (as opposed to Group 1 subjects who also had differences in sensitivity) because the points lay along the axes where a signal detectability curve could fit them. There were, of course, other Group 2 subjects with different sensitivity estimates for the four cues, but the observed ROC points were distributed further from the axes of the ROC space. As noted earlier, a straight line with variable slope would apparently fit subjects 8, 9, 10, and 11

Table 14.

Estimates of σ

Subject	Group 1				Group 2			
	C1	C2	C3	C4	C1	C2	C3	C4
1	.02	.14	.19	.05	.26	.21	.31	.40
2	.57	.66	.56	.52	.28	.35	.30	.29
3	.33	.33	.39	.24	.04	.05	.17	.01
4	.35	.44	.38	.36	.72	.78	.80	.76
5	.01	.13	-.08	.02	.26	.40	.38	.39
6	.35	.39	.40	.42	.15	.38	.18	.34
7	.86	.91	.90	.90	.22	.30	.21	.25
8	.65	.65	.60	.61	.81	.65	.57	.63
9	.46	.42	.45	.47	.35	.40	.22	.24
10	.37	.37	.19	.22	.83	.88	.72	.75
11	.10	.21	.27	.12	.54	.77	.50	.41
12	.80	.76	.83	.73	.78	.64	.78	.72
Average	.41	.45	.44	.39	.44	.48	.43	.43

in Group 2 quite well. It is reasonable that an experiment of the simple detection or recognition type should have difficulty in distinguishing between signal detectability curves and variable sensitivity theory curves, since the less biased points are assumed in both theories to be closer to (0,1) in the ROC space than are the more biased points.

In application to the present experiment, Case 1 and Case 2 essentially reduced to a fixed sample size model where

$$P(A_1|S_1) = \frac{d}{D} + (1 - \frac{d}{D})g$$

and

$$P(A_1|S_2) = (1 - \frac{d}{D})g \ .$$

Since the display size in the present experiment (16) was identical to one of the conditions in an earlier experiment by Estes and Taylor (1965), it should be interesting to compare the present estimates of d to their P, the estimated average number of elements (symbols) processed according to the serial-processing model. From Tables 6 and 7 we can see that the average d was approximately 6.5 for Group 1 and about 7 for Group 2. This is quite close to $P = 5.57$ for $D = 16$ in the Estes and Taylor experiment.

Table 15 shows that estimates of d were roughly consistent for those models that did not assume probability matching (estimates of d were not obtained for Case 3). The reason that Cases 1a and 4 yield larger estimates of d is probably that they explain the shift or asymmetry in $P(A_1)$ across the cues by means of activation variables rather than through the bias mechanism as do the other models. To the extent

Table 15.

Estimates of d

for Various Recognition-Confusion Models

Group 1

Subject	Case 1	Case 2	Case 1a	Case 1b	Case 4
1	1	1	2	1	8
2	9	9	10	7	11
3	6	5	6	6	9
4	7	6	10	7	14
5	1	1	--	1	5
6	6	6	8	6	11
7	14	14	16	14	15
8	10	10	12	10	13
9	8	7	8	7	9
10	4	4	5	1	7
11	2	2	3	2	5
12	12	12	14	14	14
Average	6.70	6.40	8.55	6.82	10.08

Group 2

Subject	Case 1	Case 2	Case 1a	Case 1b	Case 4
1	5	4	5	4	6
2	5	5	6	5	7
3	1	1	1	1	2
4	12	12	15	14	15
5	5	5	6	5	7
6	4	3	5	5	10
7	4	3	5	4	7
8	11	10	12	10	12
9	4	4	--	4	9
10	13	12	15	13	15
11	9	8	10	8	11
12	12	11	13	11	13
Average	7.10	6.83	8.46	7.55	10.18

that this shift was an important characteristic of the data, the estimates of d will differ for the two types of models.

In a different type of psychophysical experiment, Sperling (1960) and Averbach and Sperling (1961) found under stimulus conditions comparable to those in the present experiment that approximately 3/4 of the presented letters were "available" to the subjects. In the present experiment this would mean that 12 letters were available to the subjects. Although the average value of d for the best fitting three-parameter model, Case 4 (about 10), was substantially larger than d for the other cases, d is still less than 12. The probable reason for the disparity between Sperling's values and our estimated values of d is that his subjects were not required to process all 12 letters. Thus, it may be that the subject selects a sample from the available pool of symbols which he then proceeds to process. An alternative model that might do well would assume that d is equal to the number of symbols initially available but that a decay of the type postulated by Estes and Taylor (1964) sets in immediately after stimulus offset. If this were the case, d would have to be an increasing function of D, according to experiments involving different values of D performed by Estes and Taylor (1965) and Sperling (1960).

A striking facet of the data which was not commented on earlier is the increase in the sensitivity estimates (σ_h) over test sessions. Note that while this result may cause some difficulty in the exact interpretation of the estimated parameters, as long as σ changes in the same way for the different cues, this change does not affect the comparison between theories that predict straight line ROC curves and theories that predict curvilinear ROC curves. This follows from the fact that an average of straight lines is a straight line. Figures 11 and 12 were obtained using the fixed sample size model where σ_h represents $d/16$ (d was not constrained to integral values here) and the bias parameters were estimated separately for each cue and subject, and averaged over subjects in each group. Note that the increase in σ_h is not accompanied by a regression of the g_h toward 1/2 as one might predict under the variable sensitivity concept. It is also interesting that Group 1 shows an increasing shift in the bias parameters toward A_2. Support would be lent to the notion that the $P(A_1)$ asymmetry was due to an E_2 advantage over E_1, as opposed to the hypothesis that Z_2, Θ, was more confusable with the noise symbols than was Z_1, Φ, had Case 1b fit Group 1 subjects better than did Case 1a. Also, Group 2 <u>was</u> fit better by Case 1b than by Case 1a, but showed no g_1 decrease over days.

It is apparent from an examination of the bias functions for Group 2 that the g_h does not accurately reflect the experimental correlations, since $g_1 > g_2$ but $\gamma_1 < \gamma_2$ and $g_4 < g_3$ but $\gamma_4 > \gamma_3$. The reason for this failure by Group 2 to follow the schedule may be spatial generalization. The linear arrangement of the cue lights was such that C_1 and C_4 were always on the outside, but C_2 and C_3 were always on the inside. Although C_2 and C_3 were the more highly correlated cues for Group 2, their proximity and the subjects' knowledge that the two cues on either side were positively correlated with different stimulus events may have led to their failure to learn the actual cue-stimulus correlations.

The superiority of Case 4 (over the other three-parameter models considered) in explaining the ROC data is somewhat surprising in view of comments by the subjects obtained after the experiment. The prevalent

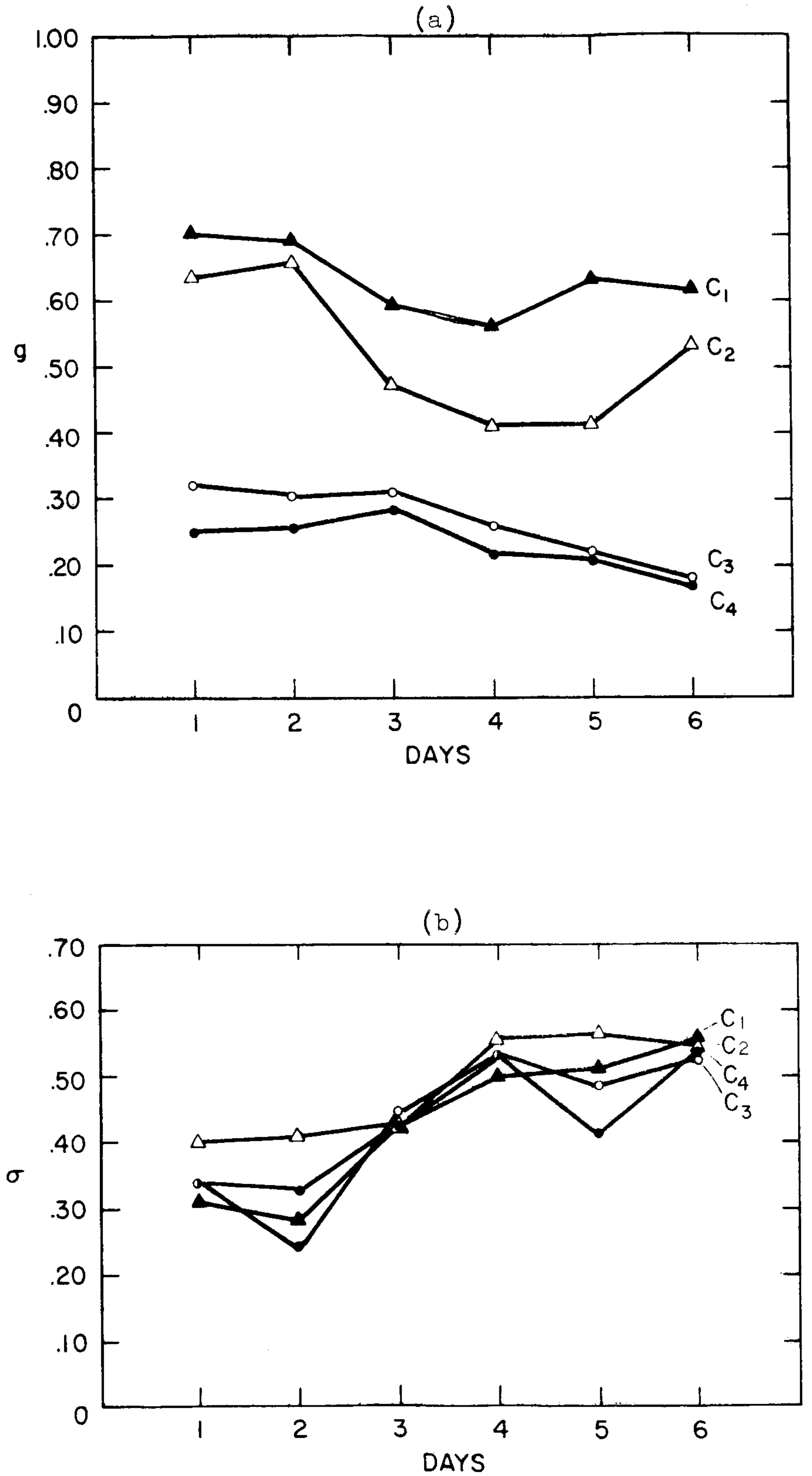

Figure 11. Estimates of g_h and σ_h as Functions of Day for Group 1.

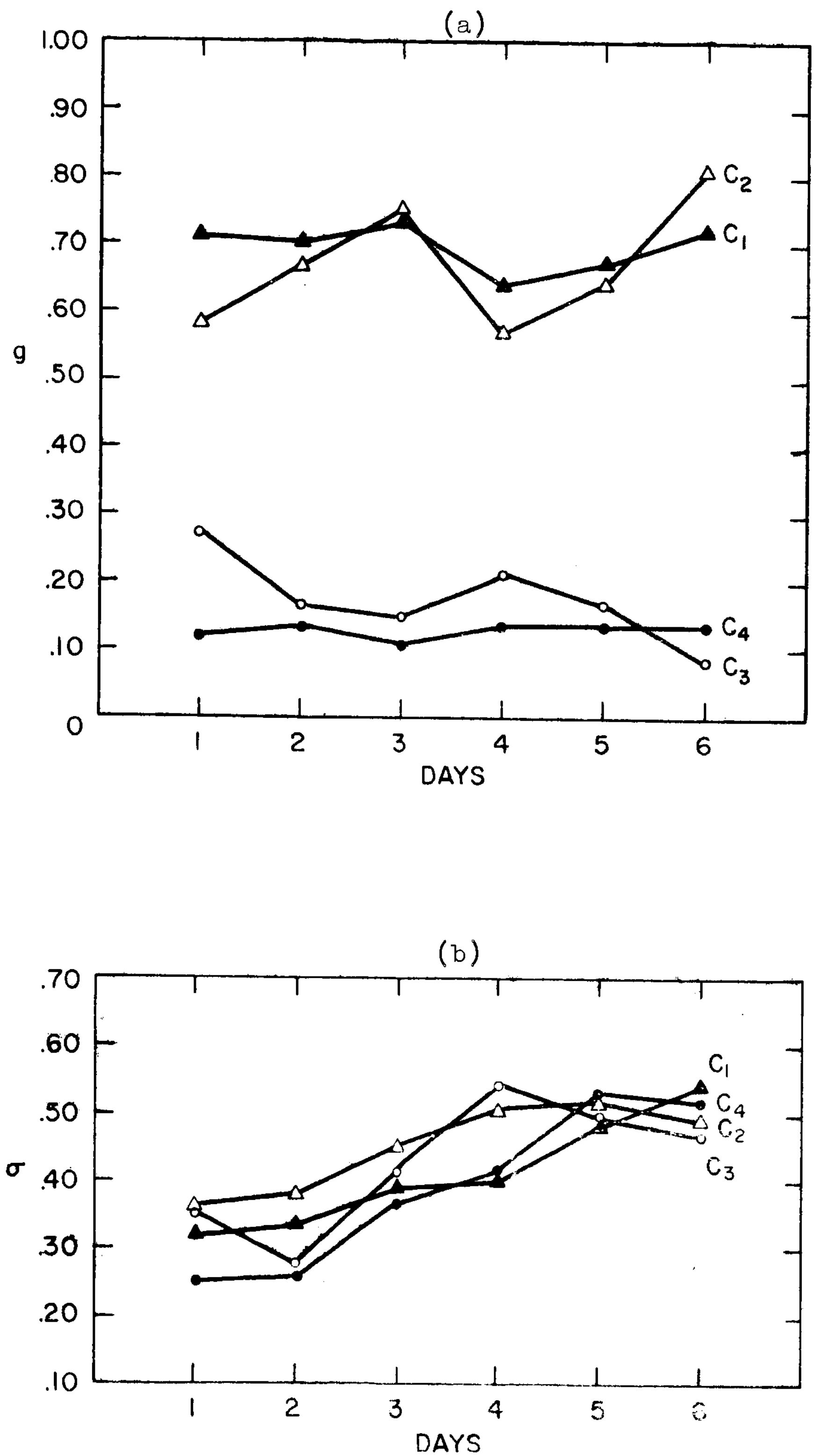

Figure 12. Estimates of g_h and σ_h as Functions of Day for Group 2.

response was that several of the noise symbols, B and G in particular, were often confused with $\ominus$ but that few if any of the noise symbols were ever confused with ⦶. The failure of Case 1a to do a better job than it did is possibly due to the untenability of the assumption that all the Z_0 symbols were alike in confusability with the signal symbols. However, one might expect that this would be remedied in the estimations by the high u value. An additional possibility that would be interesting to test is that Case 1a might do as well or better than Case 4 if g_h were treated as a free parameter. That is, the probability matching constraint may not have affected Case 1a and Case 4 to the same extent. On the other hand, since the basic form of both signals was a circle, it is reasonable that there should be confusion between $\ominus$ and ⦶, although the source of the asymmetry in confusion is not clear. The superiority of Case 4 to Case 3 is probably due to the incapacity of Case 3 to provide for the A_2 bias without increasing the slope of the ROC curve. A detailed description of the data might involve an activation matrix with entries in all the cells, but it seems likely that inter-signal confusion was a potent factor.

The remainder of the discussion will be devoted to the latencies and the confidence-rating results.

Examination of Fig. 9 leads to the conclusion that if the recognition models applied to the ROC data can fit the latencies in this experiment at all, they must do so by virtue of the τ_k included to represent the number of Δt units required to make a guessing response. This is not to imply that the model is wrong; it does say that the form of the latency functions as the guessing bias g varies is determined by τ rather than by ℓ, which predicts (for example) an increase in

A_1 latencies as g increases. This prediction is contrary to the experimental results. Even allowing a different τ for the preferred and non-preferred responses is not sufficient, since some of the latencies appear to change continuously as a function of γ (and therefore g). Since the present model does not describe how τ changes as a result of changes in g, a detailed quantitative fit would seem unwarranted. However, it is interesting to note in the present context that under the fixed sample size model, the difference in the incorrect latencies conditionalized on the occurrence on the non-preferred response and the preferred response should be simply $\tau_{p'} - \tau_p$. Estimating this difference, we obtain

$$\tau_{p'} - \tau_p = \begin{cases} 80 \text{ msec. for Group 1} \\ 136 \text{ msec. for Group 2} \end{cases} .$$

Neither of these quantities is far from the average difference of 50 msec. in non-preferred and preferred response latencies obtained recently in a probability learning study (Friedman, et al., 1964).

One reasonable alternative to the hypothesis that the negative correlation between latencies and γ is due completely to τ, is that on some proportion of trials the subject, because of eyeblink, inattention, eye tremor, etc., fails to obtain any sample at all and therefore responds at once using his guessing bias. Actually, this phenomenon was reported fairly often by the subjects.

If such trials were frequent relative to the number of trials when the subject guessed after having processed all the symbols, then the kind of latency results obtained here would be expected. This could occur only if a model that allowed a fairly high rate of inter-symbol confusion explained the data.

An explanation can be obtained for the present confidence-rating results from the recognition-confusion models by assuming that the subject partitions the time following stimulus offset into 4 successive Δt periods, or what amounts to the same thing, partitions the set of possible activation positions into 4 distinct subsets. Suppose that if an activation occurs in the most recent or first set of positions, he gives his response a rating of one; if an activation occurs in the second set, he gives it a rating of two. This continues until either an activation occurs in a position located in the last set or the subject processes all the symbols and then guesses; if either of these events occurs, he uses CR_4. The results (see Fig. 10) indicate that the subjects were able to reserve CR_4 for guessing responses. This is shown by the tendency to convergence of the $P(A_1|S_1)$ and $P(A_1|S_2)$ curves until they reach CR_4; at this place both curves move in the direction of the bias. The decrement in performance for CR_1 to CR_3 implies that the activation parameters must be a function of $i\Delta t$. For instance, Case 4 might take on the form:

$$N_i = \begin{array}{c} \\ Z_0 \\ Z_1 \\ Z_2 \end{array} \begin{array}{c} \begin{array}{ccc} s_0 & s_1 & s_2 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 \\ 0 & v_1^{i-1} & 1-v_1^{i-1} \\ 0 & 1-v_2^{i-1} & v_2^{i-1} \end{bmatrix} \end{array} .$$

If we suppose that CR_4 is reserved for guesses and that a_k is the maximum position included in CR_k then

$$P(A_1|S_1CR_k) = \frac{\sum_{a_{k-1}+1}^{a_k} v_1^{j-1}}{a_k-a_{k-1}} = \frac{v_1^{a_{k-1}}(1-v_1^{a_k-a_{k-1}})}{(1-v_1)(a_k-a_{k-1})}, \quad (k < 4)$$

$$P(A_1|S_2CR_k) = \frac{\sum_{a_{k-1}+1}^{a_k} (1-v_2^{j-1})}{a_k-a_{k-1}} = 1 - \frac{v_2^{a_{k-1}}(1-v_2^{a_k-a_{k-1}})}{(1-v_2)(a_k-a_{k-1})}, \quad (k < 4)$$

$$P(A_1|S_1CR_4) = P(A_1|S_2CR_4) = g, \qquad k = 4 \quad .$$

To obtain an idea of how this function appears, let $a_k-a_{k-1} = 4$ for all $k \leq 4$, then

$$P(A_1|S_1CR_k) = \begin{cases} \dfrac{v_1^{4(k-1)}(1-v_1^4)}{4(1-v_1)} & \text{if } k < 4 \\ g & k = 4 \end{cases}$$

$$P(A_1|S_2CR_k) = \begin{cases} 1 - \dfrac{v_2^{4(k-1)}(1-v_2^4)}{4(1-v_2)} & \text{if } k < 4 \\ g & k = 4 \; . \end{cases}$$

The qualitative form of these expressions is in line with the results and indicates that meaningful predictions for confidence ratings can be derived from the recognition-confusion models. To obtain a quantitative fit, the bounds of the partitions probably should be estimated and possibly several forms of the activation matrix considered. Note, however, that a constant N_1 cannot explain the decrement in performance that occurs as a function of k.

SUMMARY

A cued-recognition paradigm was used to investigate behavior in a psychophysical task that minimized the role of immediate memory but maximized discrimination behavior. A class of models that generates predictions for several characteristics of a subject's choice behavior was developed and applied to the ROC data for each subject. Certain models appeared to provide an accurate description of the ROC results on a group and individual basis, according to an orthogonal regression measure of goodness of fit. Table 16 summarizes the various special recognition-confusion models applied to the present data.

Cases 1 and 2, when applied to the ROC data, reduced to fixed sample size models with $N_i = I$, the identity matrix. Case 4, which assumes intersignal confusion but no signal-noise or noise-signal confusion, fit the best of the four three-parameter models applied to the ROC data. Under constraints on the g_h values, Cases 1a and 1b did not reduce to the fixed sample size models for several subjects, but estimates of u remained high, thus supporting the notion of a low average noise-signal confusion. There is an element of curvilinearity in the observed ROC points which does not appear to follow from signal detectability assumptions. This curvilinearity could be associated with a sensitivity variation caused by differences in the bias parameter of the recognition-confusion model.

The fixed sample size model correctly predicts that when $d > 1$, the incorrect latencies will be longer than the correct latencies. However, the recognition-confusion models, as they are presently formulated,

Table 16

Summary of Recognition-Confusion Models Applied to Present Data

(For all the models below, the sample S consists of d symbols sampled at random.)

Case 1	$N_i = \begin{array}{c} \\ Z_0 \\ Z_1 \\ Z_2 \end{array} \begin{array}{c} \begin{array}{ccc} s_0 & s_1 & s_2 \end{array} \\ \begin{bmatrix} u & (1-u)\frac{1}{2} & (1-u)\frac{1}{2} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$	All g_h values estimated.
Case 2	$N_i = \begin{array}{c} \\ Z_0 \\ Z_1 \\ Z_2 \end{array} \begin{array}{c} \begin{array}{ccc} s_0 & s_1 & s_2 \end{array} \\ \begin{bmatrix} v^i & (1-v^i)\frac{1}{2} & (1-v^i)\frac{1}{2} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$	All g_h values estimated.
Case 1a	$N_i = \begin{array}{c} \\ Z_0 \\ Z_1 \\ Z_2 \end{array} \begin{array}{c} \begin{array}{ccc} s_0 & s_1 & s_2 \end{array} \\ \begin{bmatrix} u & (1-u)q & (1-u)(1-q) \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$	$g_h = \gamma_h$.
Case 1b	$N_i = \begin{array}{c} \\ Z_0 \\ Z_1 \\ Z_2 \end{array} \begin{array}{c} \begin{array}{ccc} s_0 & s_1 & s_2 \end{array} \\ \begin{bmatrix} u & (1-u)\frac{1}{2} & (1-u)\frac{1}{2} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$	$g_h = \frac{\gamma_h}{\gamma_h+(1-\gamma_h)\varphi}$.
*Case 3	$N_i = \begin{array}{c} \\ Z_0 \\ Z_1 \\ Z_2 \end{array} \begin{array}{c} \begin{array}{ccc} s_0 & s_1 & s_2 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 \\ 1-a & a & 0 \\ 1-a' & 0 & a' \end{bmatrix} \end{array}$	$g_h = \frac{\gamma_h}{\gamma_h+(1-\gamma_h)\varphi}$.
Case 4	$N_i = \begin{array}{c} \\ Z_0 \\ Z_1 \\ Z_2 \end{array} \begin{array}{c} \begin{array}{ccc} s_0 & s_1 & s_2 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 \\ 0 & a & 1-a \\ 0 & 1-a' & a' \end{bmatrix} \end{array}$	$g_h = \gamma_h$.

* Case 3 appears to have four parameters, but a and d, and a' and d combine in such a way in the ROC space that essentially 2 parameters, σ_1 and σ_2 are being estimated where

$$P(A_1|S_1) = \sigma_1 + (1-\sigma_1)g ,$$

$$P(A_1|S_2) = (1-\sigma_2)g .$$

do not seem particularly helpful in explicating the finer aspects of the latency results obtained in this experiment.

It was shown that particular recognition-confusion models are capable of yielding confidence-rating predictions that are in general agreement with the data.

Estimates of the number of symbols processed by the subjects compared favorably with earlier estimates in similar experiments, and these results were discussed with regard to other methods of studying the number of symbols apprehended in a brief interval.

Appendix

Table A-1

Observed Values of $P(A_1|S_2)$ and $P(A_1|S_1)$ for the Separate Cues

	C_1		C_2		C_3		C_4	
Subject Group 1	$P(A_1\|S_2)$	$P(A_1\|S_1)$	$P(A_1\|S_2)$	$P(A_1\|S_1)$	$P(A_1\|S_2)$	$P(A_1\|S_1)$	$P(A_1\|S_2)$	$P(A_1\|S_1)$
1	.886	.910	.597	.737	.498	.696	.504	.547
2	.348	.918	.207	.865	.049	.606	.027	.541
3	.381	.713	.336	.655	.184	.578	.205	.442
4	.218	.572	.156	.599	.163	.543	.150	.506
5	.901	.918	.737	.870	.486	.418	.264	.300
6	.312	.683	.228	.631	.094	.504	.034	.493
7	.089	.951	.049	.958	.028	.924	.026	.922
8	.169	.822	.162	.813	.066	.669	.037	.644
9	.367	.822	.393	.808	.127	.573	.109	.578
10	.589	.959	.587	.963	.033	.224	.019	.236
11	.831	.930	.278	.484	.121	.385	.202	.322
12	.079	.873	.092	.848	.052	.879	.045	.767
Aver.	.431	.839	.318	.770	.159	.583	.135	.525
Group 2								
1	.600	.890	.613	.841	.082	.333	.056	.475
2	.551	.834	.484	.860	.098	.407	.093	.382
3	.921	.933	.917	.944	.038	.194	.105	.090
4	.112	.829	.086	.861	.038	.833	.034	.798
5	.614	.875	.444	.840	.045	.429	.054	.440
6	.636	.737	.323	.713	.256	.424	.180	.514
7	.567	.784	.472	.769	.140	.353	.164	.413
8	.146	.959	.297	.953	.034	.588	.030	.663
9	.442	.781	.400	.801	.056	.265	.037	.278
10	.131	.962	.086	.965	.016	.735	.015	.767
11	.420	.951	.167	.934	.032	.500	.075	.477
12	.067	.843	.306	.950	.022	.794	.011	.721
Aver.	.434	.865	.383	.869	.071	.488	.071	.502

Table A-2

Observed Values of Proportion Correct P(c)

for the Separate Cues

Group 1

Subject	C_1	C_2	C_3	C_4
1	.706	.602	.577	.509
2	.849	.836	.812	.866
3	.690	.659	.722	.706
4	.624	.697	.718	.765
5	.710	.620	.477	.631
6	.685	.688	.752	.852
7	.941	.955	.953	.961
8	.824	.823	.828	.883
9	.775	.728	.753	.812
10	.820	.742	.668	.795
11	.741	.580	.682	.678
12	.885	.872	.920	.909
Aver.	.771	.734	.738	.781

Group 2

	C_1	C_2	C_3	C_4
1	.772	.796	.862	.825
2	.743	.825	.856	.775
3	.719	.858	.884	.694
4	.844	.866	.949	.924
5	.754	.810	.901	.821
6	.643	.709	.709	.744
7	.696	.744	.811	.729
8	.932	.927	.930	.892
9	.726	.781	.879	.790
10	.939	.960	.960	.930
11	.860	.923	.923	.814
12	.865	.924	.960	.923
Aver.	.791	.844	.885	.821

Table A-3

Average Latencies for Each Subject and Cue

Group 1 Subject		$E(L\|A_1S_1)$	$E(L\|A_1S_2)$	$E(L\|A_2S_1)$	$E(L\|A_2S_2)$	$E(L)$
1	C_1	1.217	1.299	1.506	1.365	1.259
	C_2	1.284	1.340	1.448	1.315	1.328
	C_3	1.335	1.437	1.502	1.393	1.403
	C_4	1.280	1.328	1.513	1.359	1.354
2	C_1	1.116	1.275	1.438	1.134	1.153
	C_2	1.138	1.534	1.499	1.066	1.117
	C_3	1.078	1.592	1.317	1.116	1.153
	C_4	1.061	1.480	1.338	1.053	1.095
3	C_1	1.102	1.345	1.208	.985	1.130
	C_2	1.011	1.344	1.259	.996	1.103
	C_3	.922	1.228	1.301	1.065	1.090
	C_4	.858	1.306	1.102	1.007	1.050
4	C_1	.978	1.193	.956	.862	.960
	C_2	.975	1.220	.949	.838	.938
	C_3	.927	1.122	.941	.861	.915
	C_4	.973	1.103	.984	.855	.914
5	C_1	.907	.942	1.301	1.906	.964
	C_2	1.041	1.080	1.255	1.374	1.105
	C_3	1.025	1.032	1.216	1.137	1.105
	C_4	1.270	1.059	.849	1.086	1.054
6	C_1	.955	1.128	.967	.914	.964
	C_2	.916	.970	.954	1.021	.962
	C_3	.916	1.139	1.017	.910	.945
	C_4	.739	1.380	1.015	.891	.900

Table A-4

Average Latencies for Each Subject and Cue

Group 1 Subject		$E(L\|A_1S_1)$	$E(L\|A_1S_2)$	$E(L\|A_2S_1)$	$E(L\|A_2S_2)$	$E(L)$
7	C_1	1.326	1.785	1.618	1.299	1.340
	C_2	1.354	1.764	1.760	1.331	1.363
	C_3	1.348	1.915	1.697	1.325	1.355
	C_4	1.357	1.800	1.623	1.323	1.346
8	C_1	.958	1.155	1.189	.919	.989
	C_2	.930	1.271	1.262	.948	.995
	C_3	.947	1.486	1.063	.955	.988
	C_4	.967	1.277	1.153	.908	.950
9	C_1	1.318	1.736	1.714	1.397	1.422
	C_2	1.458	1.711	1.830	1.437	1.535
	C_3	1.477	1.938	1.574	1.353	1.464
	C_4	1.567	1.889	1.514	1.283	1.399
10	C_1	1.613	1.664	1.813	1.605	1.626
	C_2	1.651	1.624	1.839	1.591	1.639
	C_3	1.657	1.741	1.676	1.632	1.650
	C_4	1.700	1.824	1.676	1.634	1.649
11	C_1	1.073	1.072	1.365	1.547	1.108
	C_2	1.291	1.399	1.319	1.329	1.323
	C_3	1.285	1.377	1.257	1.258	1.270
	C_4	1.333	1.427	1.346	1.269	1.311
12	C_1	1.163	1.473	1.578	1.129	1.201
	C_2	1.158	1.465	1.538	1.157	1.204
	C_3	1.155	1.432	1.566	1.173	1.194
	C_4	1.200	1.539	1.511	1.202	1.231

Table A-5

Average Latencies for Each Subject and Cue

Group 2 Subject		$E(L\|A_1S_1)$	$E(L\|A_1S_2)$	$E(L\|A_2S_1)$	$E(L\|A_2S_2)$	$E(L)$
1	C_1	1.527	1.707	1.438	1.311	1.525
	C_2	1.469	1.691	1.668	2.161	1.538
	C_3	1.176	1.820	1.449	1.255	1.307
	C_4	1.386	1.715	1.369	1.209	1.273
2	C_1	1.296	1.495	1.513	1.259	1.346
	C_2	1.249	1.932	1.397	1.225	1.302
	C_3	1.297	1.233	1.190	1.194	1.201
	C_4	1.290	1.535	1.243	1.054	1.141
3	C_1	.967	1.028	1.163	1.319	.998
	C_2	.931	.929	1.175	1.077	.944
	C_3	1.257	1.327	.856	.820	.849
	C_4	1.260	1.295	.858	.865	.906
4	C_1	.793	1.469	1.248	.848	.882
	C_2	.795	1.307	1.168	.841	.850
	C_3	.802	1.192	1.285	.818	.837
	C_4	.839	1.464	1.252	.837	.874
5	C_1	.984	1.063	1.399	1.190	1.055
	C_2	1.005	1.041	1.286	1.183	1.058
	C_3	1.128	1.345	.936	.950	.973
	C_4	1.139	1.427	1.037	.936	.992
6	C_1	1.843	1.867	1.969	1.742	1.862
	C_2	1.848	1.946	1.939	1.830	1.873
	C_3	1.680	1.957	1.955	1.737	1.798
	C_4	1.767	1.989	1.885	1.778	1.818

Table A-6

Average Latencies for Each Subject and Cue

Group 2 Subject		$E(L\|A_1S_1)$	$E(L\|A_1S_2)$	$E(L\|A_2S_1)$	$E(L\|A_2S_2)$	$E(L)$
7	C_1	1.452	1.509	1.771	1.638	1.532
	C_2	1.507	1.665	1.844	1.584	1.589
	C_3	1.658	1.667	1.806	1.584	1.611
	C_4	1.602	1.785	1.785	1.595	1.647
8	C_1	1.403	1.740	1.533	1.316	1.401
	C_2	1.415	1.517	1.508	1.434	1.423
	C_3	1.324	1.680	1.619	1.371	1.388
	C_4	1.353	1.734	1.615	1.360	1.389
9	C_1	1.484	1.622	1.722	1.461	1.535
	C_2	1.490	1.800	1.701	1.481	1.540
	C_3	1.511	1.803	1.459	1.239	1.290
	C_4	1.554	2.186	1.408	1.251	1.327
10	C_1	1.479	2.096	1.700	1.383	1.485
	C_2	1.490	1.743	2.100	1.342	1.497
	C_3	1.468	2.138	2.141	1.346	1.386
	C_4	1.542	2.350	1.694	1.357	1.424
11	C_1	1.021	1.209	1.315	1.300	1.091
	C_2	1.071	.925	1.679	1.241	1.119
	C_3	1.018	1.418	1.122	1.033	1.047
	C_4	1.053	1.438	1.349	1.168	1.193
12	C_1	1.444	1.678	1.574	1.334	1.438
	C_2	1.358	1.607	1.872	1.467	1.397
	C_3	1.414	1.777	1.494	1.350	1.366
	C_4	1.495	1.573	1.592	1.339	1.386

Table A-7

Estimates of g for the Fixed Sample Size Model

(* indicates the bias was indeterminate due to perfect performance).

Group 1

Subject	Cue 1: 1	2	3	4	5	6	Ave.	Cue 2: 1	2	3	4	5	6	Ave.
1	.90	.91	.86	.92	.95	.93	.91	.76	.87	.73	.71	.53	.49	.68
2	.85	.87	.48	.83	.79	.78	.77	.63	.64	.59	.29	.88	.60	.60
3	.65	.62	.47	.50	.43	.73	.57	.67	.44	.47	.44	.48	.47	.49
4	.31	.24	.36	.36	.44	.27	.33	.19	.28	.16	.23	.40	.41	.28
5	1.00	.80	.94	.88	.91	.95	.91	1.00	1.00	.88	.76	.72	.91	.88
6	.70	.55	.25	.59	.37	.46	.49	.64	.51	.42	.18	.09	.35	.37
7	.59	.92	.50	.00	*	.00	.40	.53	.86	.00	.00	.00	.00	.23
8	.55	.75	.31	.00	.43	.00	.34	.60	.65	.23	.00	.25	.54	.38
9	.63	.43	.67	.73	.82	.60	.65	.70	.60	.66	.70	.60	.75	.67
10	.79	.91	.97	1.00	1.00	1.00	.94	.91	.90	.97	.92	.93	1.00	.94
11	.93	.88	.98	.95	.84	.95	.92	.31	.74	.44	.26	.06	.20	.34
12	.50	.44	.30	.00	.00	*	.25	.65	.40	.13	.45	.00	.75	.39
Ave.	.70	.69	.59	.56	.63	.61		.63	.66	.47	.41	.41	.53	

Subject	Cue 3: 1	2	3	4	5	6	Ave.	Cue 4: 1	2	3	4	5	6	Ave.
1	.68	.63	.70	.66	.52	.54	.62	.79	.54	.70	.44	.36	.33	.53
2	.11	.12	.10	.41	.00	.13	.15	.00	.05	.06	.12	.11	.06	.06
3	.52	.44	.39	.14	.18	.20	.31	.70	.51	.22	.13	.09	.17	.30
4	.41	.27	.05	.34	.10	.32	.25	.29	.23	.18	.18	.22	.30	.23
5	.56	.45	.54	.38	.46	.35	.46	.38	.72	.35	.10	.20	.03	.30
6	.07	.37	.18	.33	.09	.00	.17	.03	.06	.00	.18	.00	.05	.05
7	.57	.22	1.00	.00	*	.00	.36	.15	.25	1.00	*	*	.25	.41
8	.26	.15	.14	.12	.12	.16	.16	.05	.08	.17	.00	.06	.50	.14
9	.31	.29	.15	.13	.31	.19	.23	.20	.23	.24	.11	.25	.25	.21
10	.04	.07	.00	.03	.07	.04	.04	.07	.00	.05	.00	.00	.00	.02
11	.15	.32	.13	.22	.04	.10	.16	.25	.27	.34	.19	.12	.12	.22
12	.14	.25	.40	.42	.29	*	.34	.09	.19	.10	1.00	1.00	.00	.40
Ave.	.32	.30	.32	.26	.23	.18		.25	.26	.28	.22	.22	.17	

Table A-8

Estimates of g for the Fixed Sample Size Model

(* indicates the bias was indeterminate due to perfect performance).

Group 2

	Cue 1							Cue 2						
Subject	1	2	3	4	5	6	Ave.	1	2	3	4	5	6	Ave.
1	.87	.92	.57	.96	.79	.91	.83	.76	.71	.70	.74	.77	1.00	.78
2	.91	.83	.57	.73	.75	.82	.77	.71	.79	.84	.81	.68	.78	.77
3	.96	.88	.85	.90	1.00	1.00	.93	.95	.80	.96	.92	1.00	1.00	.94
4	.21	.00	.75	.29	.50	.00	.29	.00	.00	.62	.00	.00	.60	.22
5	.73	.94	.85	.90	.73	.85	.83	.51	.81	.69	.86	.75	.83	.74
6	.63	.67	.77	.65	.72	.88	.72	.62	.46	.46	.43	.44	.73	.52
7	.64	.78	.88	.64	.61	.78	.72	.62	.80	.76	.49	.56	.60	.64
8	1.00	.81	.75	.49	.00	.86	.65	.75	.82	.95	.90	.89	.88	.87
9	.79	.57	.64	.54	.69	.75	.66	.45	.51	.84	.67	.82	.82	.68
10	.76	.85	.75	.54	.86	1.00	.79	.00	.68	.75	1.00	.00	*	.49
11	.86	.97	.88	1.00	.86	.85	.90	.69	.69	.69	.00	.83	.81	.62
12	.21	.20	.50	.00	.54	.00	.24	.88	.93	.75	.00	.90	.90	.73
Ave.	.71	.70	.73	.64	.67	.72		.58	.67	.75	.57	.64	.81	

	Cue 3							Cue 4						
Subject														
1	.13	.10	.25	.20	.08	.00	.13	.07	.13	.20	.13	.15	.07	.13
2	.06	.09	.41	.12	.11	.28	.18	.18	.19	.17	.07	.09	.10	.13
3	.04	.10	.05	.02	.08	.00	.05	.00	.13	.09	.18	.18	.05	.11
4	1.00	1.00	.10	.05	.25	.00	.40	.25	.32	.10	.08	.10	.00	.14
5	.12	.05	.22	.09	.05	.00	.09	.03	.20	.05	.10	.07	.11	.09
6	.25	.15	.30	.64	.35	.32	.34	.29	.11	.24	.45	.28	.30	.28
7	.21	.25	.15	.25	.06	.16	.18	.18	.30	.23	.14	.13	.35	.22
8	.07	.06	.10	.10	1.00	.04	.23	.15	.09	.04	.08	.09	.00	.07
9	.18	.07	.04	.03	.05	.00	.06	.06	.11	.05	.04	.03	.00	.05
10	1.00	.06	.00	.09	.00	.00	.19	.21	.00	.00	.00	*	.00	.04
11	.12	.00	.03	.10	.00	.04	.05	.05	.04	.14	.15	.28	.48	.19
12	.05	.10	.10	1.00	.04	*	.26	.00	.00	.00	.14	.05	.13	.05
Ave.	.27	.17	.15	.22	.17	.08		.12	.13	.11	.13	.13	.13	

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